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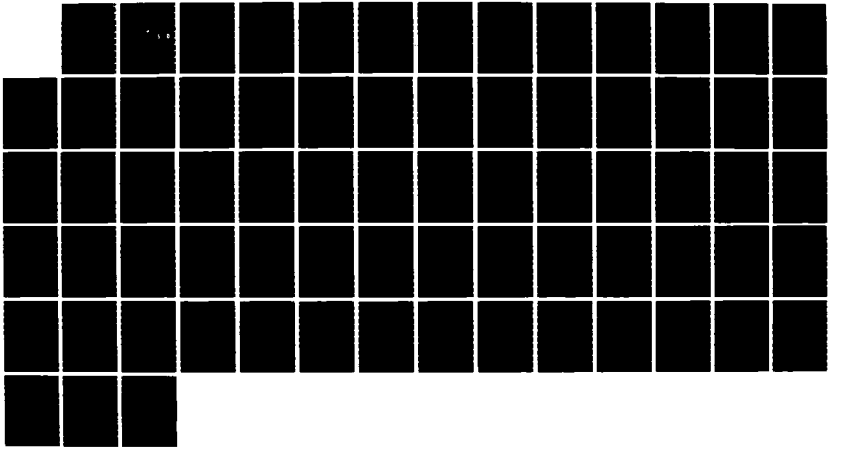

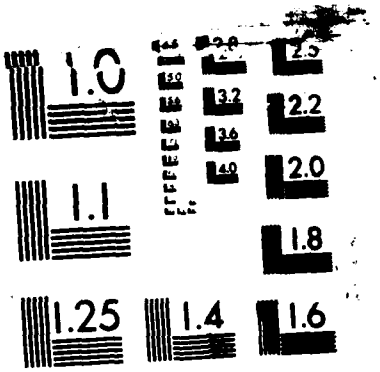

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NAVAL POSTGRADUATE SCHOOL
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THESIS

STATE-SPACE ANALYSIS OF
THE TRIDENT II MARK 6 GYRO

by

David L. Krueger

March 1986

Thesis Advisor: R. J. Collins

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State-Space Analysis Of the Trident II Mark 6 Gyro

by

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Lieutenant, United States Navy
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Submitted in partial fulfillment of the requirements for the degree of

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ABSTRACT

This thesis conducts a state-space analysis of the Mark 6 gyro used in the Trident II missile system. Optimal control techniques, utilizing a region controller, have been employed in designing a gyro controller. The response of the system to missile motion is examined and compared to the system response using the current control laws. It is shown that an optimal region controller reduces the gyro error in the system while effectively avoiding gimbal lock, a situation which exists when gyro orientation prevents missile motion from being isolated in an orthogonal three axis coordinate system. Recommendations for the application of additional modern control techniques are included.

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I. INTRODUCTION

A need for accurate, self-contained navigation systems has existed since the early days of aircraft design. The most common of these systems uses accelerometers to determine body motion. The output signal is integrated once to obtain velocity and again to obtain position.

In order for the navigation system to accurately determine body motion the sensors must be mounted on a platform fixed in a known coordinate system. In ballistic missile guidance systems, gyros are used to fix a platform in inertial space. The accelerometers are mounted on this platform. The purpose of the gyro controller, then, is to generate the gyro torque motor commands necessary to maintain the stable platform fixed in inertial space.

Since 1957, the Charles Stark Draper Laboratory in Cambridge, Massachusetts has designed the gyros for the Navy's Polaris, Poseidon and Trident missiles. Because of the complex, highly non-linear nature of the control system, this design process has been a tedious, iterative affair. Using state-space representation of systems, computer-aided optimal control design techniques greatly simplify the multi-input, multi-output non-linear time varying problem. The goal of this thesis is the development of a state-space model of the Trident II gyro system, the Mark 6, and the application of modern control theory to improve system performance.

A state-space model of the system has been developed and is compared to the current gyro model. Transient response of both systems to missile motion is examined. Modern control techniques using optimal feedback gains are employed to minimize gyro errors and improve system response. The choice of optimal feedback gains is made based on a region

controller which selects the gains dependent upon the particular gimbal angles. Finally, suggestions are made for employing modern control techniques to further optimize system performance.

II. FUNCTION OF THE GYRO CONTROLLER

A. THE ALL-ATTITUDE GIMBAL ASSEMBLY

The inertial navigation system for a ballistic missile must accurately determine missile motion in three orthogonal planes. Missile motion is resolved by accelerometers mounted on a platform fixed in inertial space. A three dimensional coordinate system is initialized before missile launch, and remains fixed through out the flight. The function of the gyro controller is to keep the stable platform fixed throughout the flight.

The Mark 6 gyro consists of four concentric gimbals as shown in Figure 2.1 [Ref. 1: p. 2]. The innermost gimbal is the stable platform, housing the accelerometers. As the missile experiences pitch, roll and yaw a torque is applied to the gimbals in such a manner as to isolate the stable member from the motion.

1. Gimbal Lock

In addition to maintaining a fixed inertial reference, the gyro controller must prevent "gimbal lock". This occurs when the gimbals are co-planar and it becomes impossible to isolate missile motion on three orthogonal axes. Controlling the gyro errors is relatively straightforward, while preventing gimbal lock is more complex. Since the gimbal angles can assume any value during missile flight, the problem becomes non-linear and difficult to solve. The technique developed by Draper Laboratory for preventing gimbal lock will be employed in the design of the state-space system model. This technique is more fully described in the next chapter.

2. Missile to Stable Member Motion Coupling

The four gimbal IMU model is shown in Figure 2.2 [Ref. 2: p. 2]. Missile, or base motion occurs in the

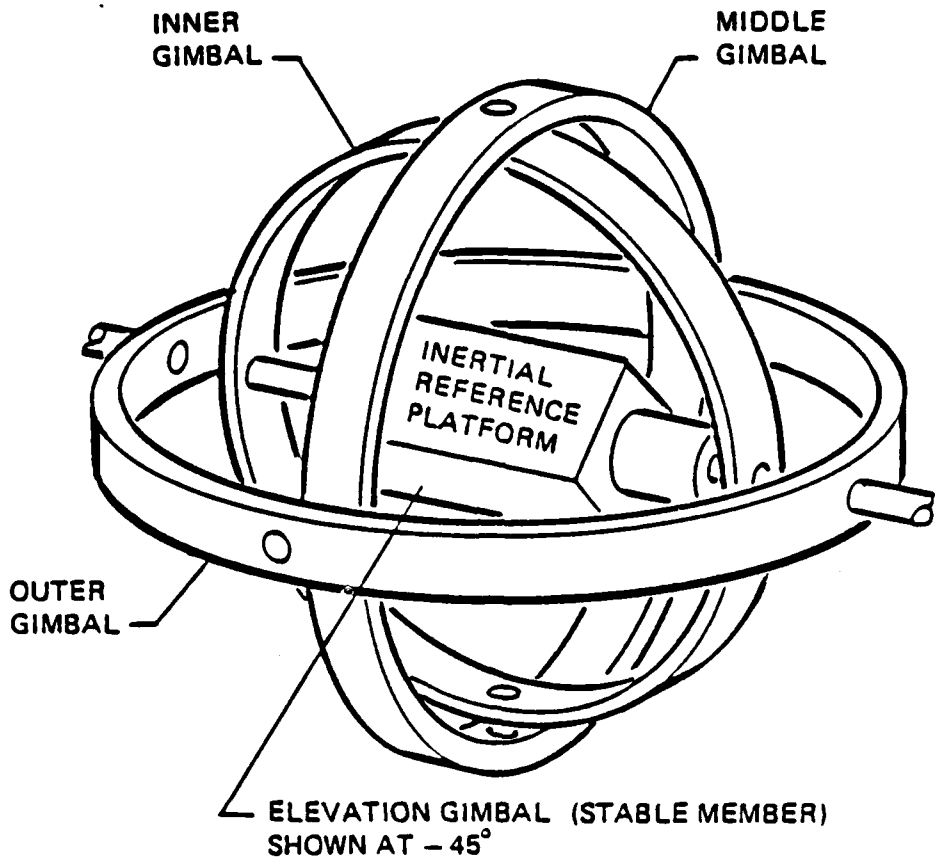


Figure 2.1 Four Gimbal Gyro Model.

i_5, j_5, k_5 coordinate system. This motion is coupled to the stable member coordinate system i_1, j_1, k_1 by the outer, middle and inner gimbals. The system accuracy is measured by the gyro error of the stable member in the i_1, j_1, k_1 space. Appendix B contains the detailed transformation of missile motion in the base frame of reference to motion in the inertial frame.

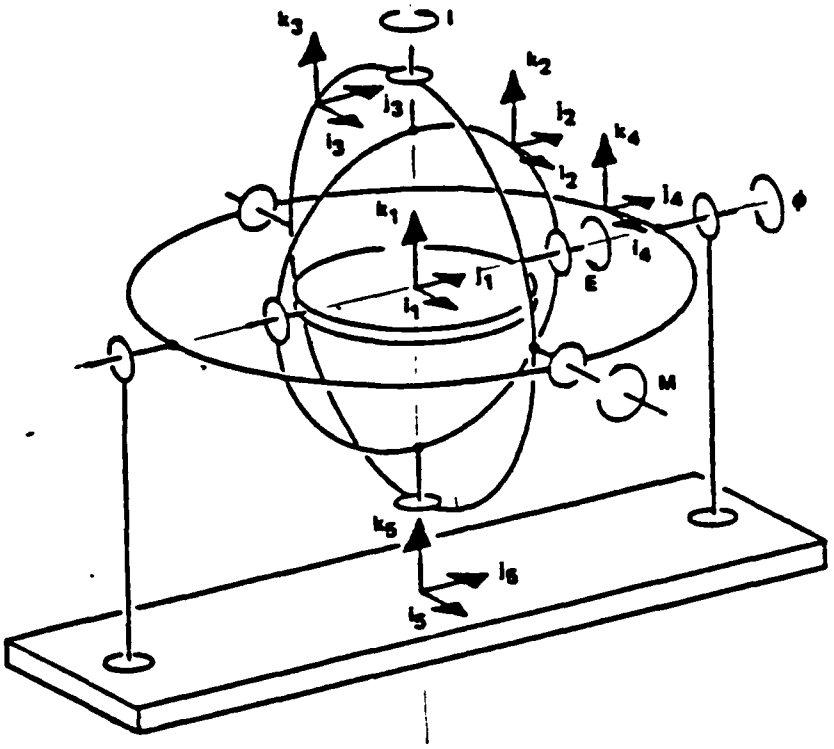


Figure 2.2 Four Gimbal Gyro Coordinate Systems.

III. MARK 6 GYRO MODEL

A. BLOCK DIAGRAM

The Mark 6 Gyro model presently being used by Draper Laboratory is shown in Figure 3.1 [Ref. 2: p. 8]. Table III in Appendix A contains a definition of abbreviations used in this thesis. The torque motor commands are the inputs to the system, with the gyro errors and gimbal angles as the output. The diagram illustrates the cross-coupling between gimbals represented by the input equations shown in Figure B.1.

1. Plant Diagram

a. Signal Conversions

Figure 3.1 contains the electrical and mechanical realizations of the Mark 6 gyro. The input signal is expressed in terms of quanta, with one quanta equal to 0.0735 volts. The torque command is expressed in ounce-inches. The gimbal angles, given in radians, are converted to quanta for input to the feedback controller.

b. RGyro

Also shown on the diagram is the RGyro. This is not a part of the plant. The RGyro defines an error associated with the roll axis. In earlier three gimbal gyros, the roll axis of the missile did not coincide with any of the gimbal axes. The four gimbal Mark 6 gyro aligns the roll axis with the outer gimbal axis, thus eliminating the need for the RGyro as an indication of gyro error. The output is still provided for ease in comparing current model errors with previous systems.

c. Gimbal Inertias

The gimbal inertias are represented in Figure 3.1 as J_S, J_I, J_M and J_O for the stable member and inner, middle and outer gimbals respectively. The outer and middle

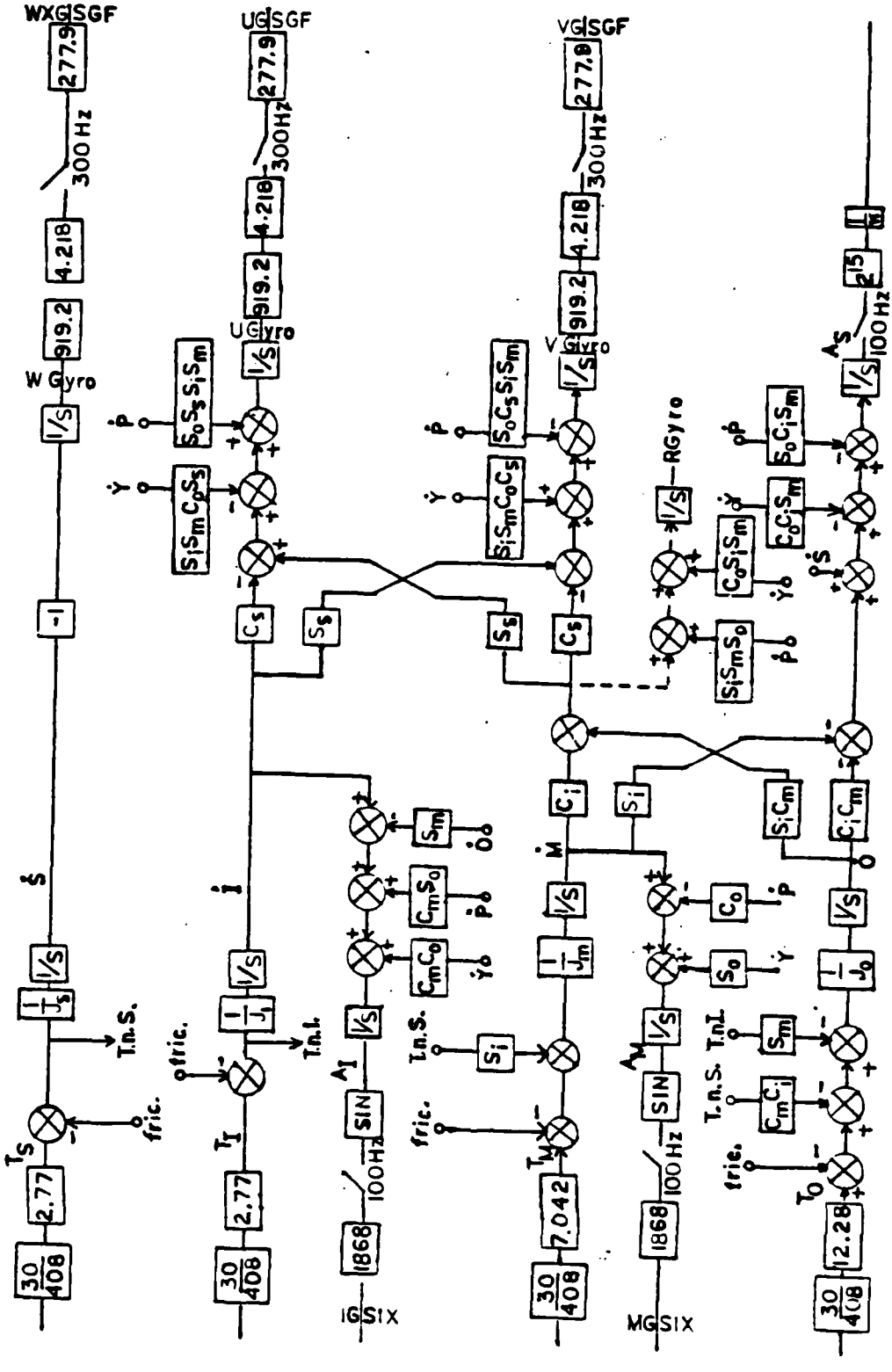
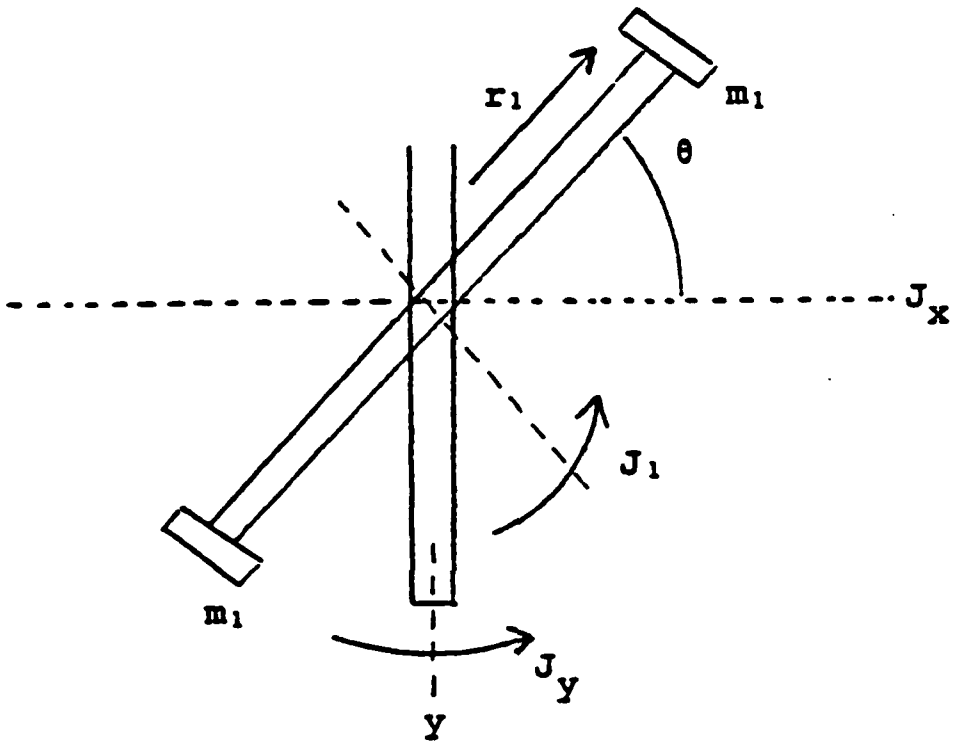


Figure 3.1 Mark 6 Gyro Model (Inertial Mode).

gimbal inertias change with time as a function of the inner gimbal angle. Figure 3.2 shows how the inertias are resolved [Ref. 2: p. 6].



$$J_1 = r_1 \cos\theta$$

resolving along the x and y axes:

$$J_x = 1_y^2 m = r_1^2 m \sin^2\theta = J_o \sin^2\theta$$
$$J_y = 1_x^2 m = r_1^2 m \cos^2\theta = J_o \cos^2\theta$$

for the four gimbal model:

$$J_M = J_M + J_I + J_S C_I^2$$
$$J_O = J_O + J_M + J_I C_M^2 + J_S C_M^2 S_I^2$$

Figure 3.2 Resolution of Outer and Middle Gimbal Inertias.

d. Stable Element

The stable member is represented by the upper flow path in Figure 3.1. There are no external inputs to the stable member, with only friction affecting the loop. The stable element is controlled by its own feedback loop and is not influenced by the other system signals. Signals from the stable loop do cross-couple to the other gimbals, however, with the torque command coupling to the middle and outer gimbals. The inertial rate S serves as an input to the stable member angle, shown at the bottom of the figure. This angle could also be shown connected to the stable element flow path, with an input from the outer gimbal inertial rate. Because the stable member is affected only by friction, it has not been included in the state-space model.

e. Sampling Rates

Two different sampling rates are used in the system. The U, V and W gyros are sampled at 300 Hz. The gimbal angles are sampled at 100 Hz.

f. Gimbal Lock Avoidance

The proximity of the system to gimbal lock can be measured by observing the sines of the inner and middle angles. The product $SIN(A_I)*SIN(A_M)$ gives an indication of how close the system is to gimbal lock, with gimbal lock occuring when the product equals one. The goal of gimbal lock avoidance is to drive the product of the sines to zero. This is accomplished by setting either the inner or middle angle to 0 or 180 degrees.

Only one angle need be controlled at a time. Figure 3.3 shows the regions where control would switch from one angle to another. This is based on the smallest of the absolute values of the sines.

The missile motion input to the U and V gyros has a gain factor of $SIN(A_M)*SIN(A_I)$, as seen in Figure 3.1 Thus, in addition to avoiding gimbal lock, nulling the

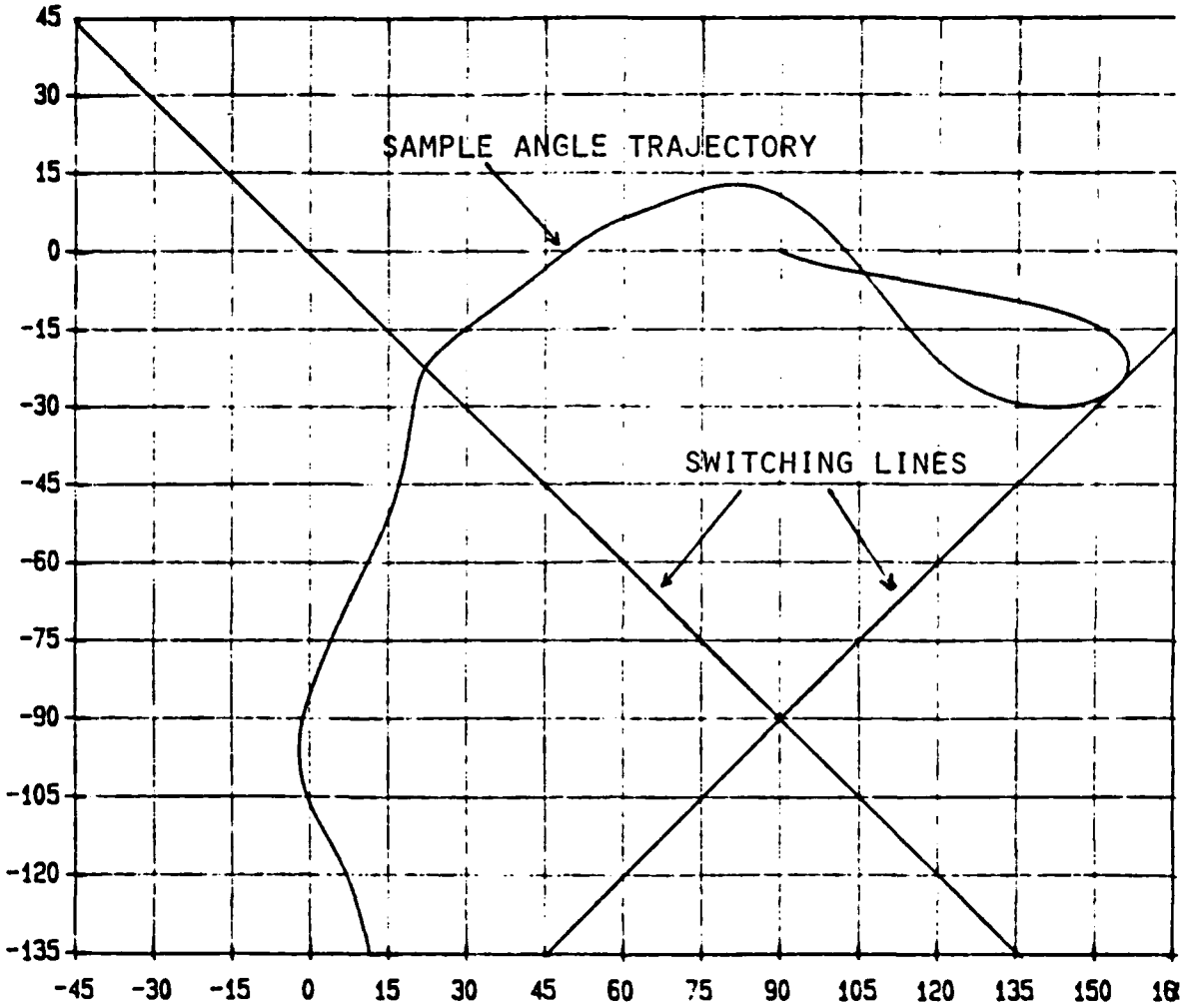


Figure 3.3 Gimbal Lock Avoidance Switching Line.

product of the sines of the inner and middle gimbal angles also minimizes the effect missile motion has on gyro error.

2. Gyro Controller

a. Control Laws

Figure 3.4 shows the controller for the Mark 6 gyro [Ref. 3: p. 115]. The six inputs to the controller are the three signals generated from the U,V and W gyros, the stable element gimbal angle and the sines of the inner and middle gimbal angles. The four outputs are the torque motor command signals.

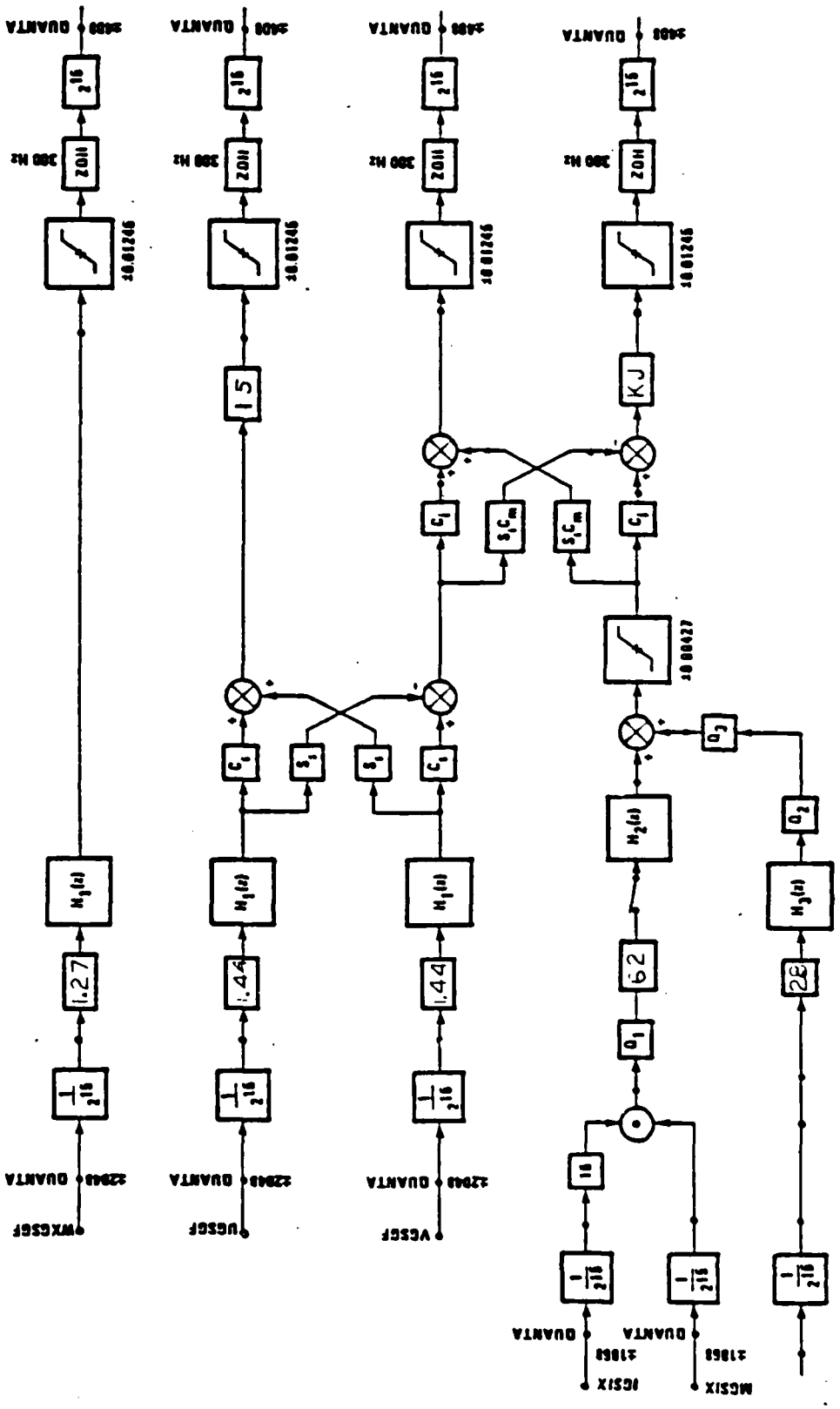


Figure 3.4 Mark 6 Gyro Control Law.

b. Transfer Functions

The transfer function $H_1(z)$ in the stable element, inner and middle gimbal feedback paths is defined as:

$$H_1(z) = (z-.932)^3(z-.80955)/(z-1)^2(z-.6186)(z-.17474)$$

The transfer functions $H_2(z)$ and $H_3(z)$ in the outer gimbal control loop are:

$$H_2(z) = (z-.9778)/(z+.3827)$$

$$H_3(z) = (z-1)/z$$

c. Redundant Torque

The outer gimbal control loop has three inputs controlled by switches Q_1,Q_2 and Q_3. Two of the inputs are the $SIN(A_M)$ and $SIN(A_I)$ signals. The third input is the stable element angle. The two signals generated by this loop are necessary when the product $SIN(A_M)*SIN(A_I)$ becomes very small and does not produce a large enough error signal to drive the outer gimbal. Q_1 and Q_3 are switches with values of +1 and -1 depending on gimbal geometry. Switch Q_2 is closed for values of $SIN(A_M)*SIN(A_I)$ less than 1/8, and open for values greater than 1/8.

d. Scaling Factors

Gain blocks with 2^{15} and $1/2^{15}$ scaling factors are included in the control laws. These gain factors are applied in order to facilitate software processing of the feedback signals only.

IV. STATE-SPACE MODEL

A. BLOCK DIAGRAM

1. Plant Model

The state-space model of the Mark 6 Gyro is shown in Figure 4.1. Where appropriate, the gains from Figure 3.1 have been combined into a single block. Only the inner, middle and outer gimbals have been modeled. Eight states are identified on the figure.

2. Assumptions and Simplifications

A number of assumptions and simplifications have been made in order to develop the state-space model:

(a) there is no friction (this eliminates the stable element loop)

(b) all states are available for feedback

(c) the inner and middle angles are fed back directly, not the sines of the angles

(d) there are no limiters in the system

(e) the entire system is sampled at 300 Hz

3. State-Space Equations

The state-space equations of the system are in the form:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$

The vectors x(t) and u(t) are described in Table I. A(t) and B(t) are the plant and control matrices, respectively. The matrices for this system are shown in Figure 4.2.

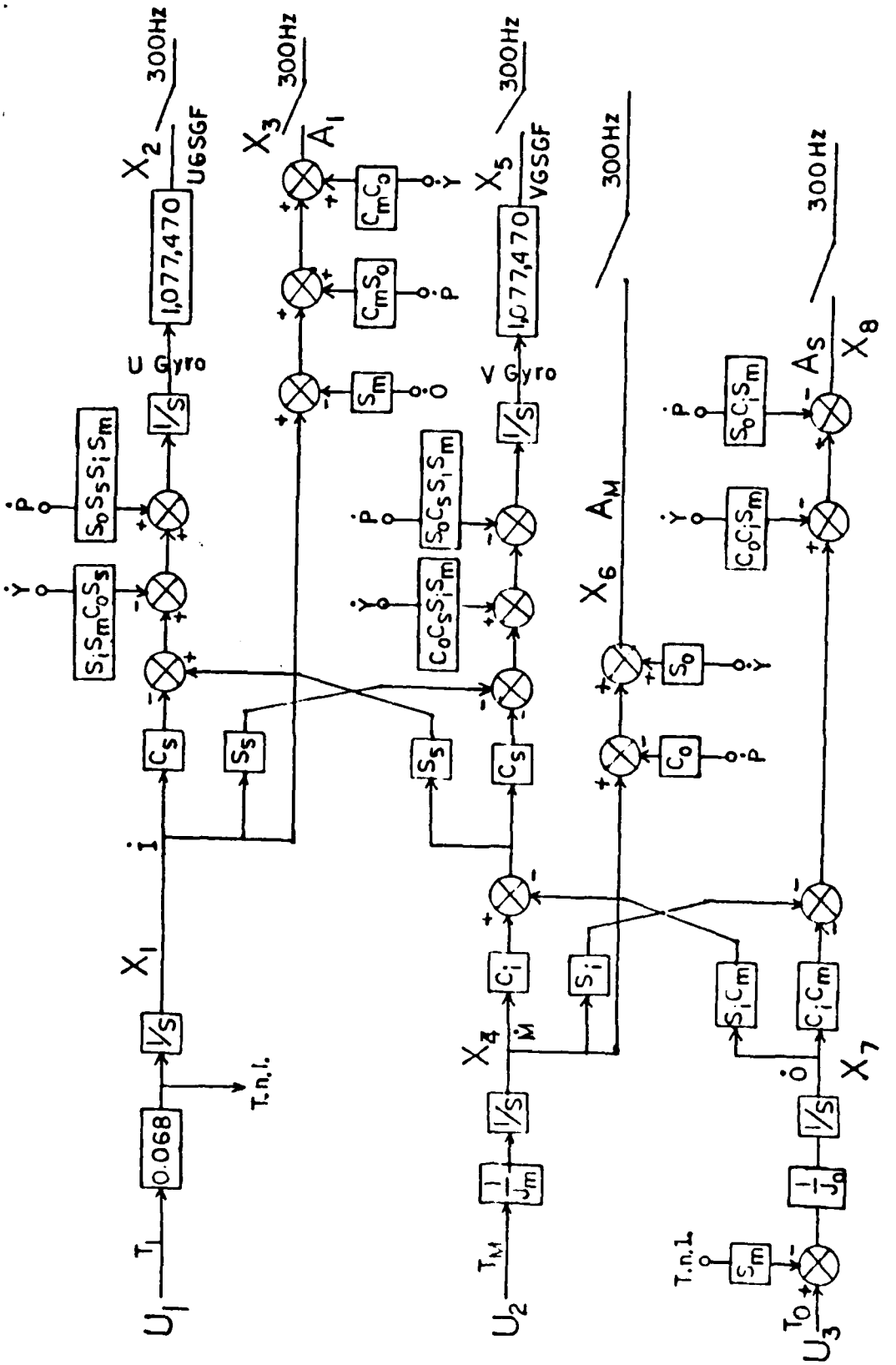


Figure 4.1 State-Space Model of the Mark 6 Gyro.

TABLE I

STATE AND CONTROL VECTORS

x_1	inner gimbal inertial rate I
x_2	U Gyro error
x_3	inner gimbal angle A_M
x_4	middle gimbal inertial rate M
x_5	V Gyro error
x_6	middle gimbal angle
x_7	outer gimbal inertial rate O
x_8	stable element gimbal angle
u_1	inner gimbal torque signal
u_2	middle gimbal torque signal
u_3	outer gimbal torque signal

Plant Matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -KC_S & 0 & 0 & KC_I S_S & 0 & 0 & -KS_I C_M S_S & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & -S_M \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ KS_S & 0 & 0 & -KC_I C_S & 0 & 0 & KC_S S_I C_M & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -S_I & 0 & 0 & -C_I C_M & 0 \end{bmatrix}$$

where K = 1,077,470

Control Matrix

$$\begin{bmatrix} 0.068 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1/J_M & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -S_M/J_O & 0 & 1/J_O \\ 0 & 0 & 0 \end{bmatrix}$$

Figure 4.2 Plant and Control Matrices for State-Space Model.

V. OPTIMIZATION TECHNIQUE

A. OPTIMAL REGULATOR

1. Linear Quadratic Problem

The Mark 6 gyro model can be represented by the state differential equation

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$

The goal of the optimization technique is to drive the system to the desired state as quickly as possible, while using only practical input amplitudes. The quadratic integral criterion can be used to express how rapid an initial state is driven to the final state [Ref. 4: p. 201]. This criterion can be expressed as:

$$J = \int x^T(t)R_3(t)x(t)dt$$

$R_3(t)$ is an appropriate weighting matrix chosen to minimize the output state vector. The criterion as represented above would require very large input amplitudes, usually unattainable in a practical system. Including the input in the quadratic integral yields

$$J = \int [x^T(t)R_3(t)x(t) + u^T(t)R_2(t)u(t)]dt$$

By proper selection of R_2 and R_3, the output vector can be minimized using realizable input amplitudes.

Since only the controlled variables are of interest, the cost equation can be written in terms of the output vector z(t), given as:

$$z(t) = D(t)x(t)$$

A weighting matrix R_1 can now be defined as:

$$R_1(t) = D^T(t)R_3(t)D(t)$$

$R_1(t)$ is a square, positive semi-definite symmetric matrix having the same order as the number of controlled variables of the system. The selection of the specific values for R_1 and R_2 are discussed in the next section.

The inputs for the Mark 6 gyro are the signals to the torque motors. These signals are generated solely as a function of gyro error and gimbal angular position. In the optimal regulator problem, the inputs can be expressed in terms of an optimal feedback gain matrix and the states.

$$u(t) = -Fx(t)$$

The state differential equation can then be written as:

$$\dot{x}(t) = A(t)x(t) + B(t)(-Fx(t)) + B(t)u_o(t)$$

When there are no external inputs, $u_o(t)=0$. Combining terms yields the following:

$$\dot{x}(t) = [A(t)-B(t)F(t)]x(t)$$

In the linear quadratic problem, the control law can be expressed as:

$$u(t) = -R_2^{-1}B^TPx(t)$$

The optimal feedback gain matrix now becomes:

$$F = R^{-1}B^TP$$

The matrix P is found by solving the Riccati equation:

$$PA + A^TP - PBR_2^{-1}B^TP + R_1 = 0$$

This is the solution for the steady-state optimal feedback gains, and is the method used to develop the feedback gain matrices used to control the Mark 6 gyro.

2. Choice of Weighting Matrices

The weighting matrices R_1 and R_2 were chosen based on the maximum values that the output vector and inputs could be expected to reach. When the weighting matrices have values only on the diagonal, the matrix product x^TRx becomes x_i^2/R_{ii} for each of the values of x. R_{ii} is then set equal to the inverse of the maximum expected value of x_i squared. This drives each of the terms in the cost equation to one when their value approaches the maximum allowed, and to zero when their value is small.

3. Non-zero Steady State Outputs

The solution to the linear quadratic optimal feedback gain problem has been outlined in the previous sections. This solution assumes, however, that the desired steady-state value of the controlled states is zero. Since the Mark 6 control law includes nulling the sine of the inner or middle angle, it is possible that the desired value of the inner or middle angle be 180 degrees, not 0. In this case, the optimal feedback gain control law becomes:

$$u(t) = -F[-R+x(t)]$$

The vector R here represents the reference signal in the system, as shown in Figure 5.1. When R=0, the optimal control solution reduces to that shown above.

In addition to an axis shift, driving the system to a non-zero set point can also be accomplished by adding a constant factor u_o to the control input u(t). The control input then becomes:

$$u(t) = -Fx(t) + u_o$$

A u_o can be chosen such that when the system experiences a shift in the set point, as the gyro controller does when shifting from 0 to 180 degrees, an optimal response is obtained while remaining within allowable input amplitudes. [Ref. 4: pp.270-275].

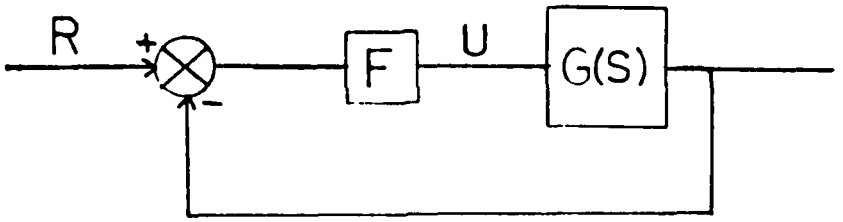


Figure 5.1 Optimal Gain Feedback Block Diagram.

This optimal constant input is defined as:

$$U_o = H_c^{-1}(0)z_o$$

In this equation, $H_c(s)$ is the closed loop transfer function from the inputs to the desired non-zero steady state output z_o. The use of this solution requires that the number of inputs is equal to the number of outputs, and that the inverse of transfer function $H_c(s)$ exists. If the transfer function has any zeros at the origin, the matrix $H_c(0)$ is singular and the inverse does not exist.

The optimal solution for the gyro controller generates one or more zeros at the origin depending on the particular gimbal angles. Thus, the optimal input u_o cannot be found and the axis shift technique must be used.

B. MARK 6 GYRO OPTIMAL CONTROLLER

1. Region Controller

With a sampling rate of 300 Hz, computing the optimal feedback gains every sampling interval would require an enormous amount of computations. Lauro has shown that a region controller will yield a nearly optimal response while greatly reducing the calculations involved [Ref. 1: p. 8].

The plant and control matrices of the Mark 6 gyro are a function of the gimbal angles, which in turn vary with time. The optimal regulator designed for the system divides the gimbal angle regions up into segments of 30 degrees or less. Optimal gains for the gimbal angles at the center of the region have been calculated. When the missile motion causes the gimbal angles to change from one region to another, the controller inserts the feedback gains calculated for that region. Figure 5.2 shows typical regional divisions for two angles. The regions are 10 degrees wide near zero, and 30 degrees wide further from the origin. The figure shows a two dimensional plot, although the actual gyro controller involves four angles in four dimensional space.

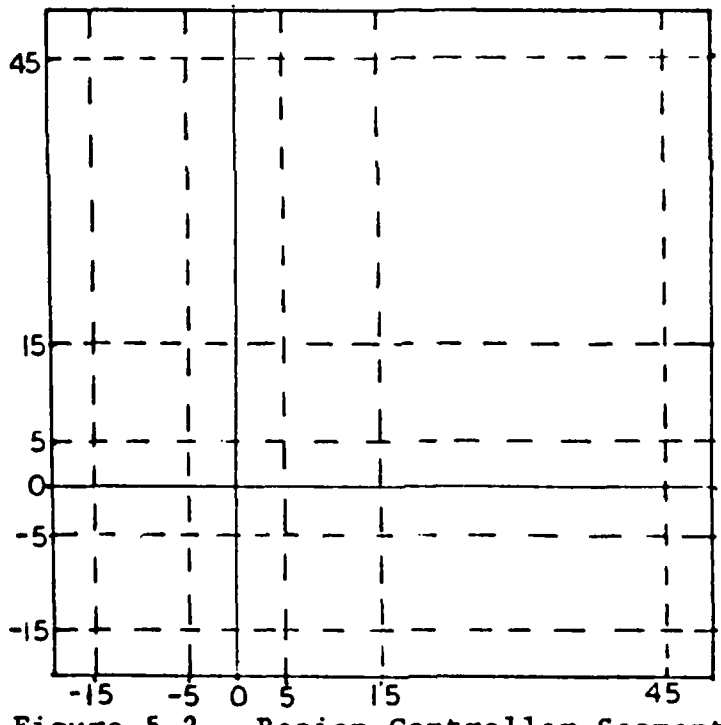


Figure 5.2 Region Controller Segments.

Figure 5.3 contains the algorithm of the computer program written to implement the optimal regulator. A copy of the program listing is included in Appendix C.

```
initialize missile motion and gyro rates to zero
assign initial gimbal angles
determine initial condition vector x(0)
     restrict all angles to +/- 360 degrees
     determine optimal gain region based on gimbal angles
     assign appropriate optimal feedback gains
     calculate variable terms in plant and control matrices
     input missile motion
     calculate x(k+1) values for each state
```

Figure 5.3 Algorithm for the Mark 6 Gyro Optimal Regulator.

VI. SYSTEM SIMULATION

A. SIMULATION MODEL

The model used for system simulation is a modified version of the state-space model shown in Figure 4.1. The U and V gyro errors are integrated, and the result nulled to minimize net gyro errors. The outer gimbal angle is calculated since it affects the plant and control matrices, but is not used for optimal feedback. The simulation model has eleven states, and is shown in Figure 6.1. The plant and control matrices for the simulation model are given in Figures 6.2 and 6.3.

B. WEIGHTING MATRICES

1. Torque Limits

Each axis of the gyro is driven by a torque motor. The amount of torque delivered by the motor is directly proportional to the feedback error signal. When the gyro errors are zero and the sine of the selected gimbal angle has been driven to null, no torque is generated.

The maximum torques which can be supplied to each axis are

- maximum inner axis torque T_{Imax} 83 oz-in
- maximum middle axis torque T_{Mmax} 211 oz-in
- maximum outer axis torque T_{Omax} 369 oz-in

Up to 14 percent more torque is available dependent upon the particular gimbal angles. This additional torque was not assumed to be available when designing the state-space system.

2. State Output Weighting Matrix R_1

The optimal control cost function is defined as

$$J = \int [z^T(t)R_1(t)z(t) + u^T(t)R_2(t)u(t)]dt$$

The output measurement vector z(t) is

$$z(t) = [x_2(t)\ x_3(t)\ x_5(t)\ x_6(t)\ x_{10}(t)\ x_{11}(t)]^T$$

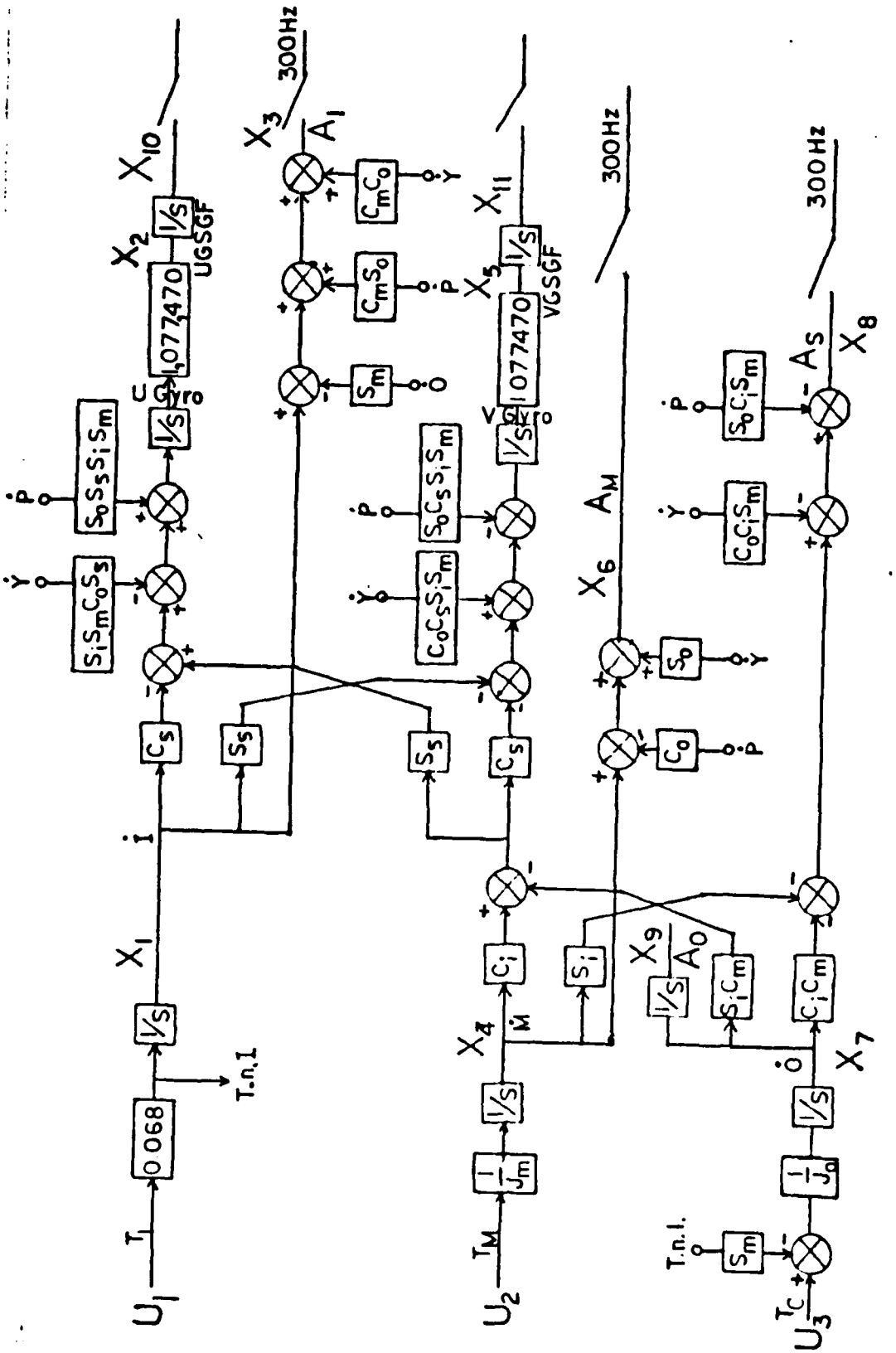


Figure 6.1 State-Space Simulation Model of the Mark 6 Gyro.

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-KCS & 0 & 0 & KCISS & 0 & 0 & -KSICMSS & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & -SM & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
KSS & 0 & 0 & -KCICS & 0 & 0 & KCSSICM & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -SI & 0 & 0 & -CICM & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$K = 1{,}077{,}470$

Figure 6.2 Plant Matrix for the State-Space Simulation Model.

$$\begin{bmatrix} 0.068 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1/J_M & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -S_M/J_O & 0 & 1/J_O \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Figure 6.3 Control Matrix for the State Space Simulation Model.

The output vector weighting matrix was chosen based on data provided by Draper Laboratory. The output vector varies with time depending on gimbal angles. The states which comprise the output vector and their maximum allowable values are shown in Table II.

TABLE II
OUTPUT STATE VECTOR MAXIMUM VALUES

x_2	U Gyro	300 quanta
x_3	inner gimbal angle	30 degrees
x_5	V Gyro	300 quanta
x_6	middle gimbal angle	30 degrees
x_{10}	V Gyro error integral	1
x_{11}	U Gyro error integral	1

The maximum value of 300 quanta listed for the gyro angles corresponds to an error of 0.278 milliradians. This

is the maximum error attained with the current Mark 6 gimbal model. It was desired to allow a smaller error in the state-space model. The gyro has a physical stop at 2048 quanta, or 1.9 milliradians. If the error reaches this value the system will fail.

The inner and middle gimbal angle maximum values were chosen as 30 degrees. This prevents the product $SIN(A_M)SIN(A_I)$ from exceeding 0.5.

The maximum value of the integrals of the U and V gyro errors was chosen arbitrarily as 1. This limit minimized gyro error without producing excessive input amplitudes.

The R_1 matrix in terms of the output variables is

$$R_1 = \begin{bmatrix} 1/(x_2)_o^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/(x_3)_o^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/(x_5)_o^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/(x_6)_o^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/(x_{10})_o^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/(x_{11})_o^2 \end{bmatrix}$$

Using the maximum values of the output states as given in Table II, The output cost matrix becomes

$$R_1 = \begin{bmatrix} 1.11\text{x}10^{-5} & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.65 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.11\text{x}10^{-5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.65 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Control Input Weighting Matrix R_2

The control input weighting matrix is a third order symmetric matrix with the inverse of the square of the allowable torques, given previously, on the diagonal. The matrix is shown below. The inputs are related to the torques as follows:

- u_1 = inner axis torque T_I
- u_2 = middle axis torque T_M
- u_3 = outer axis torque T_O

$$R_2 = \begin{bmatrix} 1/(T_I)^2 & 0 & 0 \\ 0 & 1/(T_M)^2 & 0 \\ 0 & 0 & 1/(T_O)^2 \end{bmatrix}$$

Using the maximum torque values given previously, the R_2 matrix is:

$$R_2 = \begin{bmatrix} 1.45\text{x}10^{-4} & 0 & 0 \\ 0 & 2.25\text{x}10^{-5} & 0 \\ 0 & 0 & 7.34\text{x}10^{-6} \end{bmatrix}$$

C. TEST PARAMETERS

1. Initial Conditions

The initial conditions used for the model simulation were provided by Draper Laboratory. This particular combination of initial conditions, along with the missile motion, has been determined to be the "worst case" situation for gyro control.

The gimbal angles have been initialized as follows:

- stable member gimbal angle - 0 degrees
- inner gimbal angle - 90 degrees
- middle gimbal angle - 0 degrees
- outer gimbal angle - 59.25 degrees

The system is started from rest, with all gimbal and angular rates zero. The initial condition vector z(0) is:

$x_1(0) = 0$
$x_2(0) = 0$
$x_3(0) = 90$ degrees
$x_4(0) = 0$
$x_5(0) = 0$
$x_6(0) = 0$
$x_7(0) = 0$

$x_8(0) = 0$

$x_9(0) = 59.25$ degrees

$x_{10}(0) = 0$

$x_{11}(0) = 0$

2. Missile Motion

The missile motion used for the simulation is a pitch rate input. The input is applied at 8 rad/sec^2 to a steady-state value of 1.571 rad/sec, or 90 deg/sec. The input reaches steady-state after 0.19 seconds.

D. TEST RESULTS

The system was initialized using the values above, and the pitch input applied. The tabulated data for the simulation run is included in Appendix D.

Figure 6.4 shows a plot of U and V gyro error as a function of time. The switching points between optimal gain regions are shown on the figure. The error graphs show a transient error spike when the optimal gains are changed. The amplitude of this spike can be made arbitrarily small by reducing the size of the optimal control regions. If optimal gains were calculated and applied every sample interval, the curve would show no such fluctuations.

A comparision between the errors of the Draper lab Mark 6 model and the state-space model with optimal control region gains is shown in Figures 6.5 and 6.6. The UGSGF error signal, Figure 6.5, closely approximates the Draper model with the exception of the switching point transients. The error damps rapidly after the optimal gain change, maintaining a small steady state error. The pitch input to the U gyro has a gain including both the $SIN(A_M)*SIN(A_I)$ term and a $SIN(A_S)$ term, as seen in Figure 4.1. Since both the middle and stable gimbal angles are initially zero, the pitch rate does not affect the U gyro significantly at the start of the simulation run.

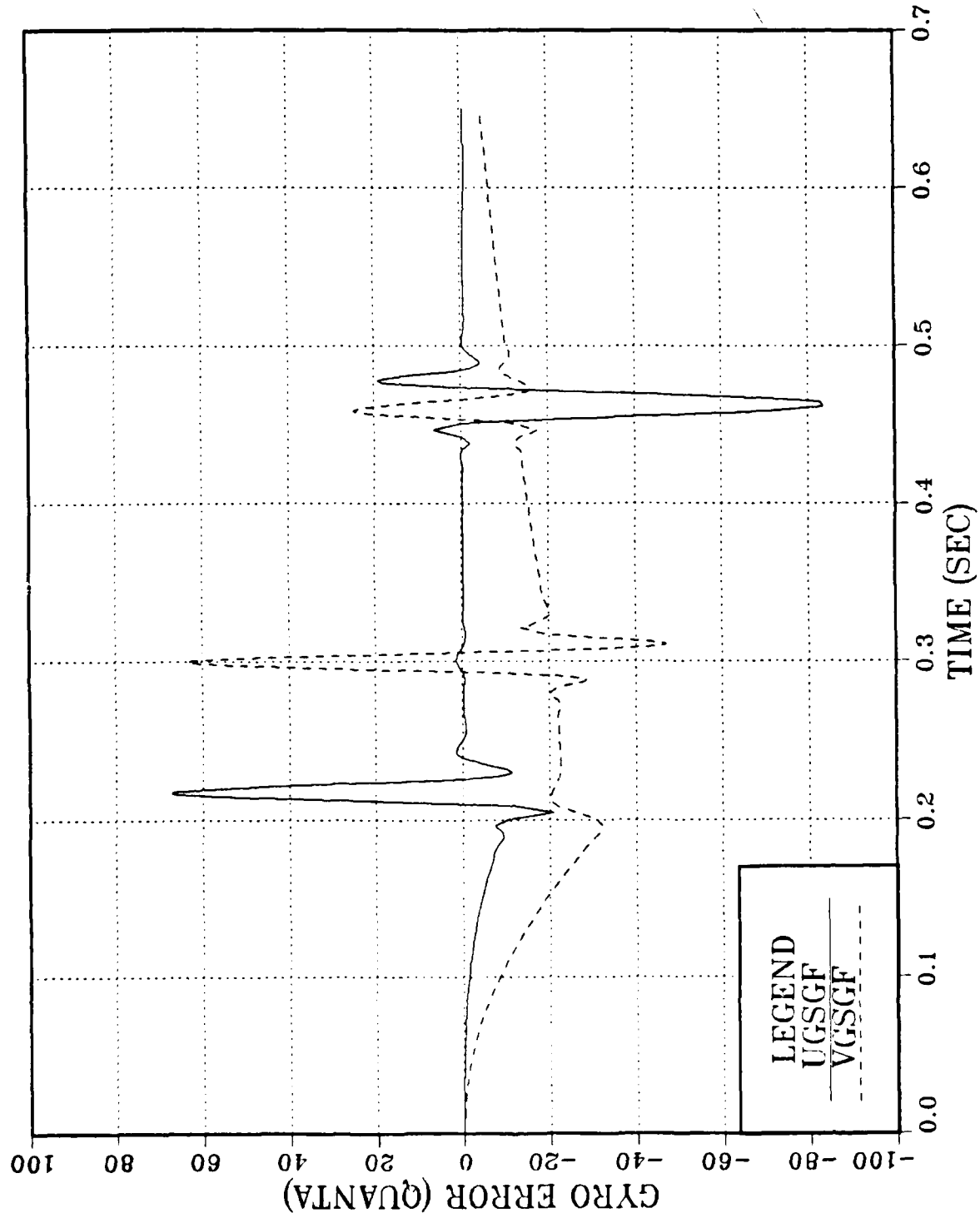


Figure 6.4 U and V gyro Errors for the State-Space Model.

The VGSGF error signal graph, Figure 6.6, shows a less pronounced response to missile motion than that experienced by the Draper model. The error transients again damp out quickly, with the system approaching but not attaining zero error at the end of the simulation run. The V gyro shows an immediate response to missile motion since the gain block contains a $COS(A_S)$ term, allowing the pitch rate to affect the gyro error as soon as the middle gimbal angle moves away from zero.

The maximum gyro errors of the Draper lab model and the optimal model, neglecting the transient error spikes, are

- U gyro error Draper lab model 12 quanta
- U gyro error optimal model 10 quanta
- V gyro error Draper lab model 94 quanta
- V gyro error optimal model 31 quanta

The maximum transient errors of the optimal model are

- U gyro error 83 quanta
- V gyro error 63 quanta

Figure 6.7 compares the inner versus middle gimbal angle of the Draper model and the state-space model. The state-space model exhibits a more rapid middle gimbal angle change than the Draper model. The angular rate of the middle gimbal angle could be slowed down by varying the maximum allowed value of the angle in the R_2 weighting matrix. A slower angular rate would result in a smaller middle gimbal angle, and the $SIN(A_M)$ term in the gain path would reduce the effect the missile motion had on the U and V gyro errors. A trade-off between the two must be made, however, since the input signals required to hold the middle angle at a slower rate may adversely affect the gyro errors.

The inner gimbal angles of both systems translate at approximately the same rate. This can be expected since the sine of the middle angle, not the inner angle, is being nulled at the start of the simulation. The inner gimbal angle is not being controlled.

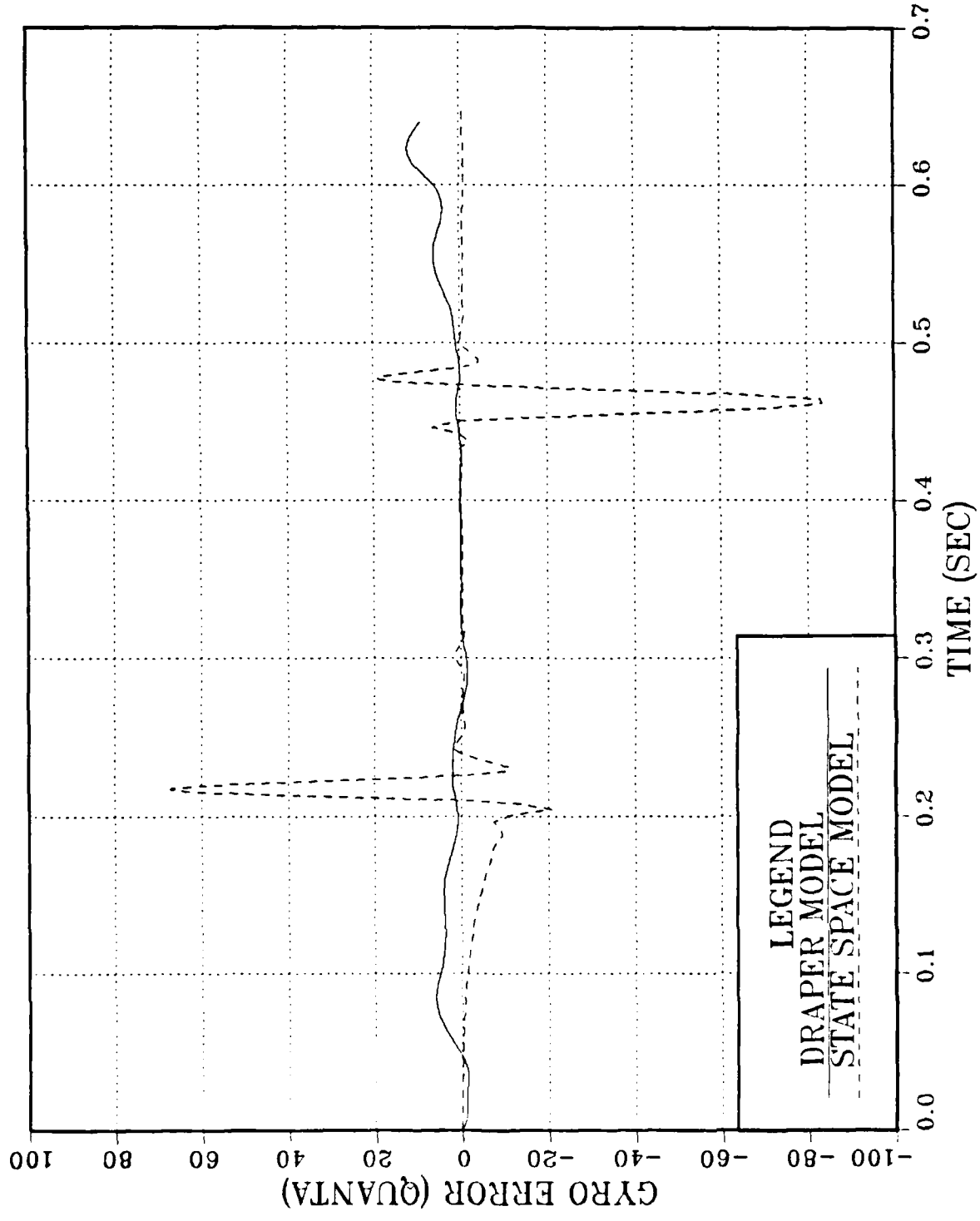


Figure 6.5 UGSGF Errors - Draper Lab and State-Space Models.

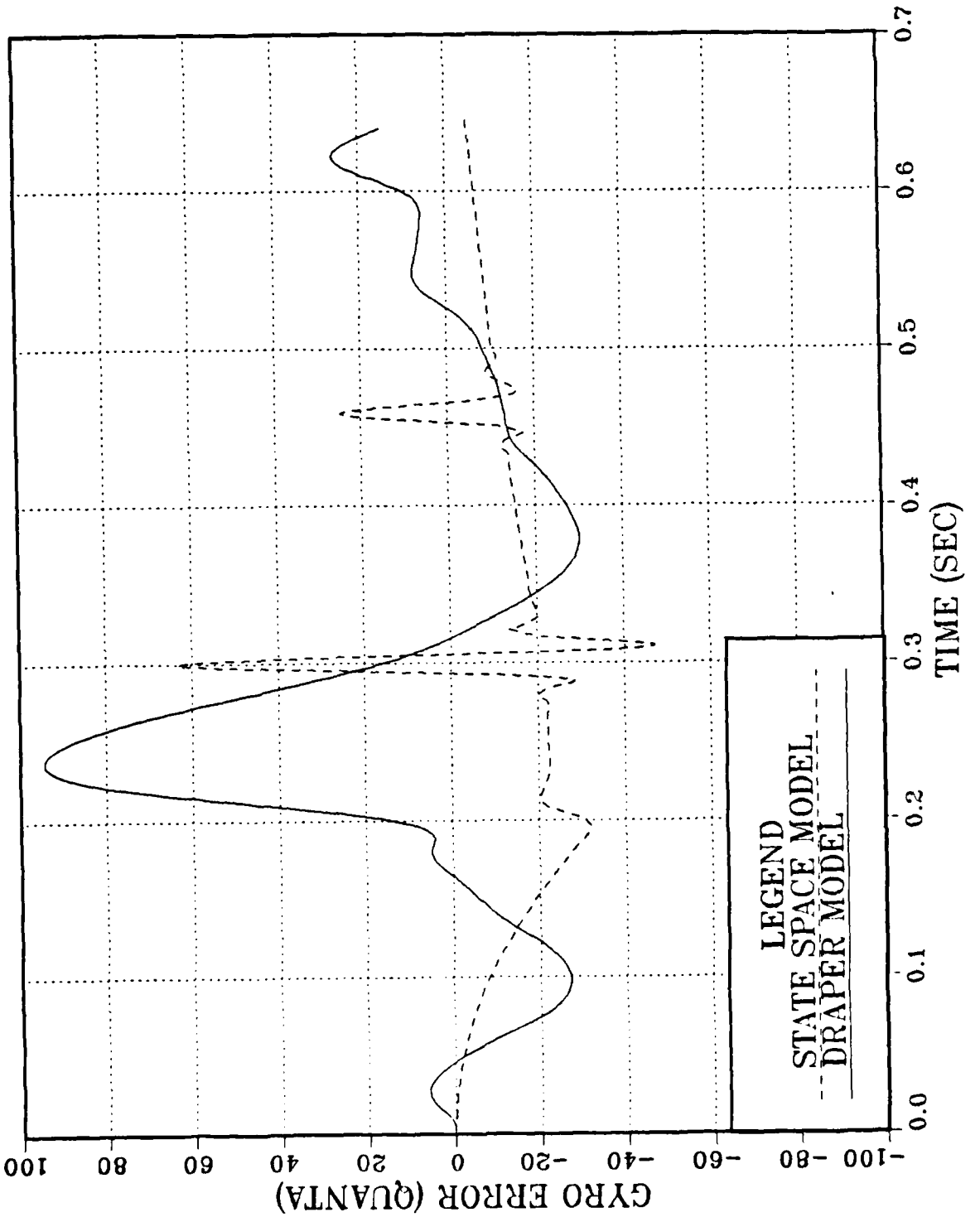


Figure 6.6 VGSGF Errors - Draper Lab and State-Space Models.

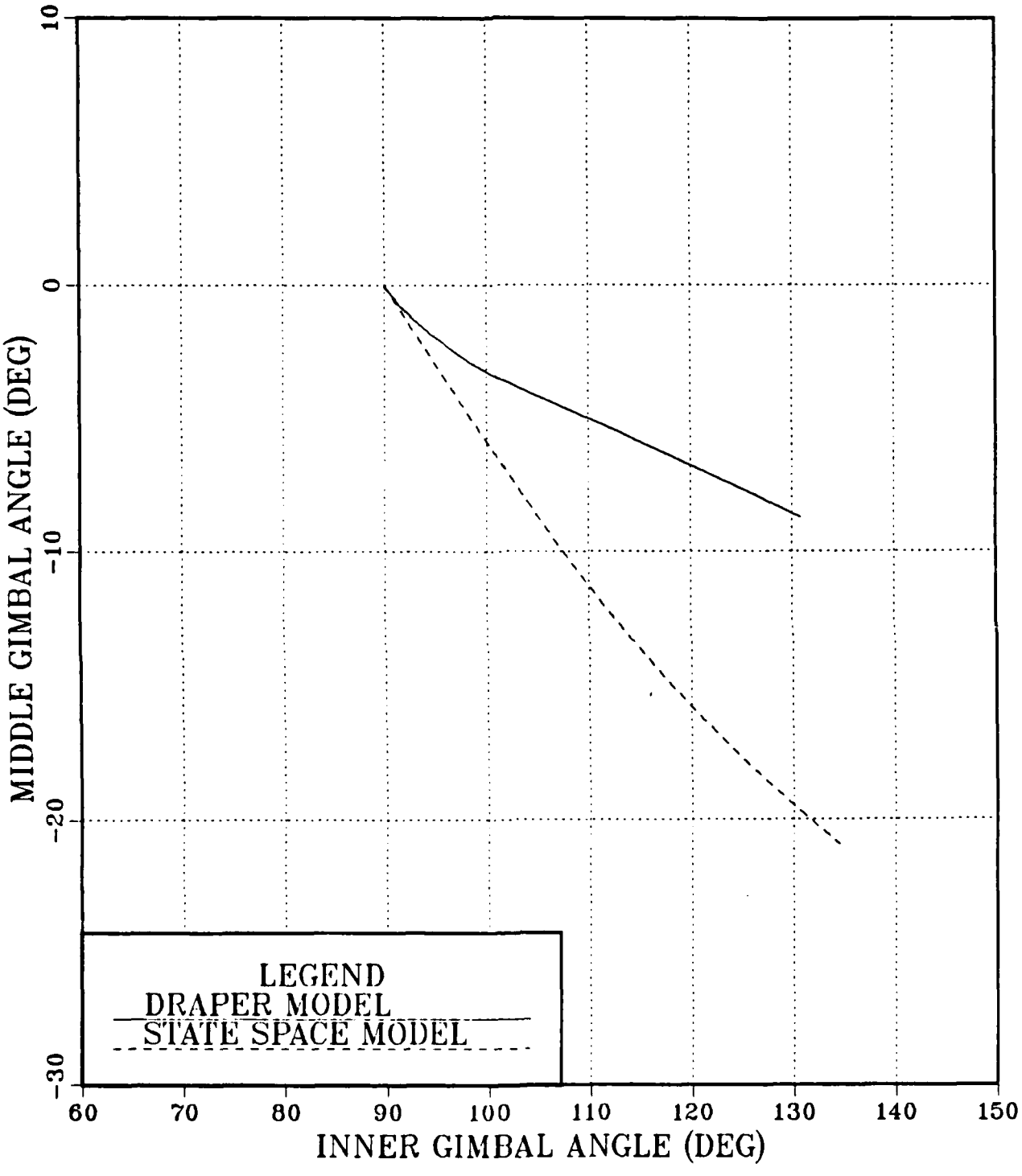


Figure 6.7 Inner and Middle Gimbal Angles.

VII. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

1. Optimal Design Techniques

It has been shown that an optimal feedback gain controller can successfully control gyro errors and prevent gimbal lock. The unique, non-linear nature of the Mark 6 Gyro requires that several modifications be made to the linear quadratic problem solution.

As the gimbal angles vary, the internal plant gains vary. Depending on the gyro orientation, these gains can range from zero to one. Care must be taken in the regions surrounding zero gain, since the loop is effectively broken at this point. If the optimal control regions chosen are too large, the instantaneous switching of the gain from zero to a finite value may cause large transient errors from which the system is unable to recover. Narrowing the optimal control regions near the zero points will lessen the impact of the loop suddenly being closed. Increasing the number of control regions would also increase the memory size required to store the additional gain values. There exists a trade-off between system performance and the amount of control hardware required to implement the system.

The constantly changing gimbal angles also lead to numerous set point shifts. It was shown in chapter five that there exists an optimal control input which drives the state to the desired value without exceeding the input amplitude limitations. This solution requires that the inverse of the closed loop transfer function exists and that the number of controlled states is equal to the number of inputs. [Ref. 4: p. 279]. If either of these conditions is not met, a set point axis shift must be used, which may cause large transient errors. Narrowing the optimal control

regions in these areas may improve system performance. Again, additional memory space would be required.

B. RECOMMENDATIONS

1. Improving System Performance

The gyro controller used in the simulation was designed using the linear quadratic optimal feedback gain design technique. Since the technique provides optimal control only at the design point, a region controller will not provide optimal response to missile motion throughout the region. Changing the weighting matrices R_1 and R_2 for a particular region may provide improved system performance while still remaining within the allowable torque limits. Since the maximum allowable value for the controlled gimbal angle and gyro error integrals are essentially arbitrary, they can be varied and the resulting system performance analyzed to determine if the error in that region can be reduced.

The simulation model also neglected the effects of friction, which can reduce net torque to the axes by 3 to 4 oz-in. The friction effects, which are a function of gimbal angular rate, may produce a marked decrease in system performance in areas where the angles are changing rapidly.

Finally, the model assumed that all states were available for feedback and that there was no noise input. In practical applications, this assumption is invalid. A Kalman filter must be designed for the system to provide an estimate of the states which can not be measured, and to optimize system performance in the presence of both process and measurement noise. Harmonics from the gyros and the torque motors may have an effect on the system, along with structural noise generated during missile flight. The non-linearity and time varying nature of the system make the design of a filter particularly complex.

APPENDIX A
DEFINITIONS

TABLE III
VARIABLE DEFINITIONS

A_S	stable element gimbal angle
A_I	inner gimbal angle
A_M	middle gimbal angle
A_O	outer gimbal angle
$\dot{S}$	stable member inertial rate
$\dot{I}$	inner gimbal inertial rate
$\dot{M}$	middle gimbal inertial rate
$\dot{O}$	outer gimbal inertial rate
S_S	sine of stable element angle
C_S	cosine of stable element angle
S_I	sine of inner gimbal angle
C_I	cosine if inner gimbal angle
S_M	sine of middle gimbal angle
C_M	cosine of middle gimbal angle
S_O	sine of outer gimbal angle
C_O	cosine of outer gimbal angle
UGSGF	U gyro error signal expressed in quanta
VGSGF	V gyro error signal expressed in quanta
IMU	Inertial Measurement Unit
	the gyro and its controller

APPENDIX B

A

1. FOUR GIMBAL IMU MODEL

This appendix develops the equations which govern the gyro motion due to inputs from missile motion and torque motors. A block diagram of the plant is developed from the equations, which also leads to the state-space model [Ref. 2: pp. 2-7].

1. Assumptions and Approximations

This analysis includes the following assumptions and approximations:

- Spherical inertias
- Small torques due to gyroscopic effects

2. Definitions

Gimbal motion is positive in a clockwise direction. The transformation from an inner member to an outer member is then in the form:

$$\begin{matrix}\text{inner member} \\ C \\ \text{outer member}\end{matrix} = \begin{bmatrix} \text{COS} & -\text{SIN} & 0 \\ \text{SIN} & \text{COS} & 0 \\ \text{COS} & 0 & 1 \end{bmatrix}$$

- Standard i,j,k axes will be used.
- x, y and z are rate components on the i, j and k axes.
- p, q and r are body, or missile, motion rates on the i, j and k axes representing pitch, roll and yaw respectively.
- A positive torque T applied to a gimbal causes a positive, clockwise rotation.
- Ex, Ey and Ez are the gyro errors sensed on the stable, or inertial, member.

3. Symbols

- $\dot{S}$ - Stable member gimbal rate resulting from applied torque
- $\dot{I}$ - Inner member gimbal rate resulting from applied torque
- $\dot{M}$ - Middle member gimbal rate resulting from applied torque
- $\dot{O}$ - Outer member gimbal rate resulting from applied torque
- $\dot{A}s$ - Stable member gimbal rate resulting from relative motion between gimbals
- $\dot{A}i$ - Inner member gimbal rate resulting from relative motion between gimbals
- $\dot{A}m$ - Middle member gimbal rate resulting from relative motion between gimbals
- $\dot{A}o$ - Outer member gimbal rate resulting from relative motion between gimbals

4. Transformation of missile motion to inertial member

a. Base motion

Base motion Wb(5) = pi5 + qj5 + rk5

referred to outer gimbal: (degree of freedom along j4 and j5)

$$Wb\Big|_{O} = \begin{bmatrix} Co & 0 & -So \\ 0 & 1 & 0 \\ So & 0 & Co \end{bmatrix} \begin{matrix} p \\ q \\ r \end{matrix}$$

$$\begin{matrix} Wbx \\ Wby \\ Wbz \end{matrix}\Bigg|_{O(4)} = \begin{bmatrix} Co*p - So*r \\ q \\ So*p + Co*r \end{bmatrix}$$

Rates along Wby do not couple to outer gimbal. O is a rate applied to the outer gimbal from its torque motor.

$$\begin{matrix} Wox \\ Woy \\ Woz \end{matrix}\Bigg|_{O(4)} = \begin{bmatrix} Co*p - So*r \\ 0 \\ So*p + Co*r \end{bmatrix}$$

b. Outer gimbal to middle gimbal transformation

Referred to middle gimbal: (degree of freedom along the i3 and i4 axes)

$$\begin{matrix} Wox \\ Woy \\ Woz \end{matrix}\Bigg|_{M(3)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & Cm & -Sm \\ 0 & Sm & Cm \end{bmatrix} \begin{bmatrix} Co*p - So*r \\ 0 \\ So*p + Co*r \end{bmatrix}$$

$$= \begin{bmatrix} Co*p - So*r \\ Cm*0 - Sm(So*p + Co*r) \\ Sm*0 + Cm(So*p + Co*r) \end{bmatrix}$$

Rates along Wox|m do not couple to middle gimbal. M is the rate of middle gimbal applied by its torque motor.

$$\begin{matrix} Wmk \\ Wmy \\ Wmz \end{matrix}\Bigg|_{M(3)} = \begin{bmatrix} M \\ Cm*0 - Sm*(So*p + Co*p) \\ Sm*0 + Cm*(So*p + Co*r) \end{bmatrix}$$

c. Middle gimbal to inner gimbal transformation.

Referred to inner gimbal (degree of freedom along k2 and k3 axes):

$$\begin{matrix} Wmx \\ Wmy \\ Wmz \end{matrix}\Bigg|_{I(2)} = \begin{bmatrix} Ci & -Si & 0 \\ Si & Ci & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M \\ Cm*O - Sm*(So*p + Co*r) \\ Sm*O + Cm*(So*p + Co*r) \end{bmatrix}$$

$$= \begin{bmatrix} Ci*m - Si*"Cm*O - Sm*(So*p + Co*r)' \\ Si*m + Ci*ECm*O - Sm*(So*p + Co*r)T \\ Sm*O + Cm*(So*p + Co*r) \end{bmatrix}$$

Rates along Wmz|I do not couple into inner gimbal. I is rate on inner gimbal applied by torque motor, therefore:

$$\begin{matrix} Wix \\ Wiy \\ Wiz \end{matrix}\Bigg|_{I(2)} = \begin{bmatrix} Ci*M - Si*ECm*O - Sm*(So*p + Co*r)T \\ Si*M + Ci*ECm*O - Sm*(So*p + Co*r)T \\ I \end{bmatrix}$$

d. Inner gimbal to stable member transformation.

Referred to stable member (degree of freedom along j1 and j2 axes):

$$\begin{matrix} Wix \\ Wiy \\ Wiz \end{matrix}\Bigg|_{S(1)} = \begin{bmatrix} Cs & 0 & -Ss \\ 0 & 1 & 0 \\ Ss & 0 & Cs \end{bmatrix} \begin{bmatrix} Wix \\ Wiy \\ Wiz \end{bmatrix}$$

$$= \begin{bmatrix} Cs*Wix - Ss*Wiz \\ Wiy \\ Ss*Wix + Cs*Wiz \end{bmatrix}$$

Rates along Wiy|s do not couple across. E is rate on stable member applied by motor, therefore:

$$\begin{vmatrix} Wex \\ Wey \\ Wez \end{vmatrix}_S = \begin{bmatrix} Cs*[Ci*M - Si*[Cm*O - Sm*(So*p + Co*r)]] - Ss*I \\ E \\ Ss*[Ci*M - Si*[Cm*O - Sm*(So*p + Co*r)]] + Cs*I \end{bmatrix}$$

e. Four Gimbal Block Diagram.

The following expressions are used to develop the four gimbal IMU block diagram shown in Figure B.1:

$$\dot{A}_E = W_{EY}j_1 - W_{IY}j_2$$

$$\dot{A}_I = W_{IZ}k_2 - W_{MZ}k_3$$

$$\dot{A}_M = W_{MX}i_3 - W_{OX}i_4$$

$$\dot{A}_O = W_{OY}j_4 - W_{BY}j_5$$

$$E_Y = Ej_1$$

$$E_X = W_{EX}i_1$$

$$E_Z = W_{EZ}k_1$$

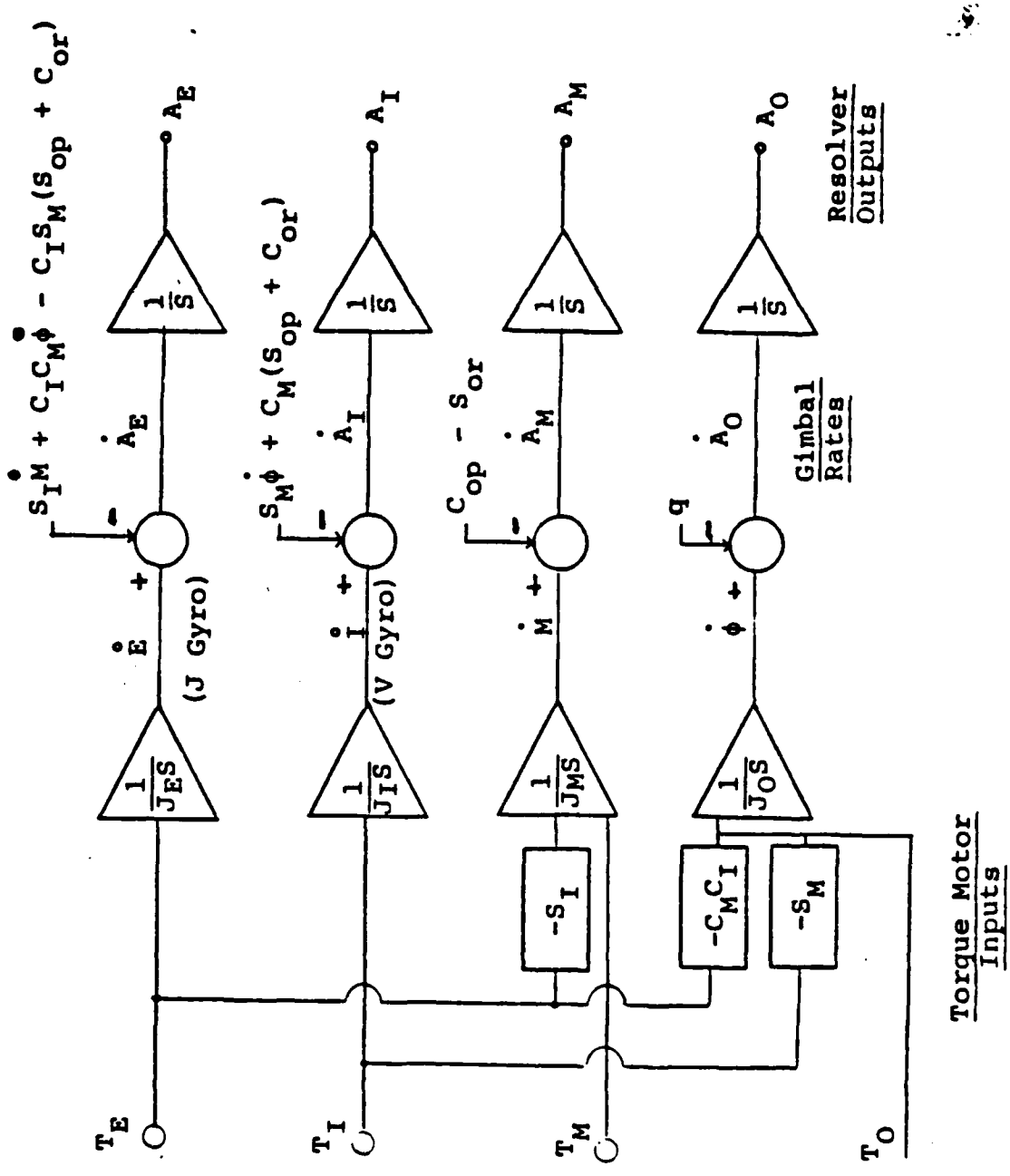


Figure B.1 Four Gimbal Gyro Block Diagram.

APPENDIX C OPTIMAL CONTROL PROGRAM LISTING

```
C     ****************************************************************  OPT00010
CCC   *                                                              *  OPT00020
CCC   *                                                              *  OPT00030
CCC   *          THIS PROGRAM CALCULATES THE TIME RESPONSE           *  OPT00040
CCC   *          OF THE MARK 6 GYRO PLANT MODEL USING TIME           *  OPT00050
CCC   *                  VARYING OPTIMAL GAINS                       *  OPT00060
CCC   *                                                              *  OPT00070
CCC   *                                                              *  OPT00080
CCC   ****************************************************************  OPT00090
CCC                                                                      OPT00100
CCC                                                                      OPT00110
CCC   DEFINE VARIABLES                                                   OPT00120
C                                                                        OPT00130
      IMPLICIT REAL*8(A-Z)                                               OPT00140
      INTEGER I,J,K,TEST                                                 OPT00150
C                                                                        OPT00160
     DIMENSION X1(3000),X2(3000),X3(3000),X4(3000),X5(3000),             OPT00170
    2X6(3000),X7(3000),X8(3000),X9(3000),X10(3000),X11(3000)             OPT00180
C                                                                        OPT00190
CCC                                                                      OPT00200
CCC                                                                      OPT00210
CCC   ASSIGN INITIAL VALUES                                              OPT00220
CCC                                                                      OPT00230
C                                                                        OPT00240
      GAIN=919.23*4.218*277.88                                           OPT00250
      PI=3.141592654                                                     OPT00260
      ANGI=90*PI/180                                                     OPT00270
      ANGM=0                                                             OPT00280
      ANGS=0                                                             OPT00290
      ANGO=59.25*PI/180                                                  OPT00300
C                                                                        OPT00310
      CS=COS(ANGS)                                                       OPT00320
      SS=SIN(ANGS)                                                       OPT00330
      SM=SIN(ANGM)                                                       OPT00340
      CM=COS(ANGM)                                                       OPT00350
      SI=SIN(ANGI)                                                       OPT00360
      CI=COS(ANGI)                                                       OPT00370
      Z=SM*SI                                                            OPT00380
      CO=COS(ANGO)                                                       OPT00390
      SO=SIN(ANGO)                                                       OPT00400
C                                                                        OPT00410
CC    CALCULATE ANGULAR MOMENTS OF INERTIA                               OPT00420
C                                                                        OPT00430
      JI=14.74176                                                        OPT00440
      JM=(0.07781+0.04949*CI**2)*192                                     OPT00450
      JO=(0.09318+0.02687*CM**2+0.04428*CM*2*SI**2)*192                  OPT00460
C                                                                        OPT00470
C                                                                        OPT00480
```

```
C       ASSIGN INITIAL CONDITION VECTOR                                  OPT00490
        X1(1)=0                                                          OPT00500
        X2(1)=0                                                          OPT00510
        X3(1)=ANGI                                                       OPT00520
        X4(1)=0                                                          OPT00530
        X5(1)=0                                                          OPT00540
        X6(1)=ANGM                                                       OPT00550
        X7(1)=0                                                          OPT00560
        X8(1)=ANGS                                                       OPT00570
        X9(1)=ANGO                                                       OPT00580
        X10(1)=0                                                         OPT00590
        X11(1)=0                                                         OPT00600
C                                                                        OPT00610
CC                                                                       OPT00620
C                                                                        OPT00630
        P=0                                                              OPT00640
        T=0.000333333                                                    OPT00650
        TWOPI=6.283185308                                                OPT00660
        PI=3.14159                                                       OPT00670
C                                                                        OPT00680
CC      STEP THROUGH ROUTINE 2800 TIMES                                  OPT00690
CC                                                                       OPT00700
C                                                                        OPT00710
        DO 10 K=1,2800                                                   OPT00720
C                                                                        OPT00730
CC                                                                       OPT00740
CC      RESTRICT GIMBAL ANGLES TO BETWEEN -360 AND +360                  OPT00750
CC                                                                       OPT00760
C                                                                        OPT00770
1       IF (X3(K).GE.TWOPI) THEN                                         OPT00780
           X3(K)=X3(K)-TWOPI                                             OPT00790
           IF (X3(K).GE.TWOPI) GO TO 1                                   OPT00800
        ELSE IF (X3(K).LE.-TWOPI) THEN                                   OPT00810
           X3(K)=X3(K)+TWOPI                                             OPT00820
           IF (X3(K).LE.-TWOPI) GO TO 1                                  OPT00830
        END IF                                                           OPT00840
2       IF (X6(K).GE.TWOPI) THEN                                         OPT00850
           X6(K)=X6(K)-TWOPI                                             OPT00860
           IF (X6(K).GE.TWOPI) GO TO 2                                   OPT00870
        ELSE IF (X6(K).LE.-TWOPI) THEN                                   OPT00880
           X6(K)=X6(K)+TWOPI                                             OPT00890
           IF (X6(K).LE.-TWOPI) GO TO 2                                  OPT00900
        END IF                                                           OPT00910
3       IF (X8(K).GE.TWOPI) THEN                                         OPT00920
           X8(K)=X8(K)-TWOPI                                             OPT00930
           IF (X8(K).GE.TWOPI) GO TO 3                                   OPT00940
        ELSE IF (X8(K).LE.-TWOPI) THEN                                   OPT00950
           X8(K)=X8(K)+TWOPI                                             OPT00960
```

```
            IF (X8(K).LE.-TWOPI) GO TO 3                                    OPT00970
         END IF                                                             OPT00980
4        IF (X9(K).GE.TWOPI) THEN                                           OPT00990
            X9(K)=X9(K)-TWOPI                                               OPT01000
            IF (X9(K).GE.TWOPI) GO TO 4                                     OPT01010
         ELSE IF (X9(K).LE.-TWOPI) THEN                                     OPT01020
            X9(K)=X9(K)+TWOPI                                               OPT01C30
            IF (X9(K).LE.-TWOPI) GO TO 4                                    OPT01040
         END IF                                                             OPT01050
C                                                                           OPT01060
C        PUT ANGLES IN DEGREES FOR OPTIMAL GAIN SELECTION                   OPT01070
C                                                                           OPT01080
         ANGS=X8(K)*180/PI                                                  OPT01090
         ANGI=X3(K)*180/PI                                                  OPT01100
         ANGM=X6(K)*180/PI                                                  OPT01110
         ANGO=X9(K)*180/PI                                                  OPT01120
C                                                                           OPT01130
C                                                                           OPT01140
C        ASSIGN OPTIMAL GAINS BASED ON GIMBAL ANGLES                        OPT01150
C                                                                           OPT01160
C***************************************************************************OPT01170
C        ROUTINE TO DETERMINE IF SIN AI OR SIN AM SHOULD BE NULLED          OPT01180
C                                                                           OPT01190
         R=0                                                                OPT01200
         TEST=0                                                             OPT01210
         TEMP1=ABS(SM)                                                      OPT01220
         TEMP2=ABS(SI)                                                      OPT01230
         IF(TEMP2.LT.TEMP1) THEN                                            OPT01240
            TEST=1                                                          OPT01250
            GO TO 444                                                       OPT01260
         ENDIF                                                              OPT01270
C***************************************************************************OPT01280
C                                                                           OPT01290
         F110=0                                                             OPT01300
         F210=0                                                             OPT01310
         F310=0                                                             OPT01320
C                                                                           OPT01330
         IF (ANGO.LT.45) GO TO 999                                          OPT01340
         IF (ANGO.GE.45 .AND. ANGO.LE.75) GO TO 1000                        OPT01350
         IF (ANGO.GT.75) GO TO 999                                          OPT01360
C                                                                           OPT01370
C***************************************************************************OPT01380
1000     IF (ANGS.LT.-45) GO TO 999                                         OPT01390
         IF (ANGS.GE.-45 .AND. ANGS.LT.-15) GO TO 1200                      OPT01400
         IF (ANGS.GE.-15 .AND. ANGS.LE.15) GO TO 1100                       OPT01410
         IF (ANGS.GT.15) GO TO 999                                          OPT01420
C                                                                           OPT01430
C***************************************************************************OPT01440
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C                                                                       OPT01450
1100  IF (ANGM.LT.-45) GO TO 999                                        OPT01460
      IF (ANGM.GE.-45 .AND. ANGM.LT.-22.5) GO TO 1140                   OPT01470
      IF (ANGM.GE.-22.5 .AND. ANGM.LT.-15) GO TO 1130                   OPT01480
      IF (ANGM.GE.-15 .AND. ANGM.LT.-5) GO TO 1120                      OPT01490
      IF (ANGM.GE.-5 .AND. ANGM.LE.5) GO TO 1110                        OPT01500
      IF (ANGM.GT.5) GO TO 999                                          OPT01510
C                                                                       OPT01520
C***********************************************************************OPT01530
C                                                                       OPT01540
1110  IF (ANGI.LT.75) GO TO 999                                         OPT01550
      IF (ANGI.GE.75 .AND. ANGI.LE.105) GO TO 1111                      OPT01560
      IF (ANGI.GE.105 .AND. ANGI.LT.135) GO TO 1112                     OPT01570
      IF (ANGI.GT.135) GO TO 999                                        OPT01580
C                                                                       OPT01590
C***********************************************************************OPT01600
C                                                                       OPT01610
1120  IF (ANGI.LT.75) GO TO 999                                         OPT01620
      IF (ANGI.GE.75 .AND. ANGI.LT.105) GO TO 1122                      OPT01630
      IF (ANGI.GE.105 .AND. ANGI.LT.135) GO TO 1121                     OPT01640
      IF (ANGI.GE.135) GO TO 999                                        OPT01650
C                                                                       OPT01660
C***********************************************************************OPT01670
C                                                                       OPT01680
1130  IF (ANGI.LT.105) GO TO 999                                        OPT01690
      IF (ANGI.GE.105 .AND. ANGI.LT.135) GO TO 1131                     OPT01700
      IF (ANGI.GE.135 .AND. ANGI.LT.165) GO TO 1132                     OPT01710
      IF (ANGI.GE.165) GO TO 999                                        OPT01720
C                                                                       OPT01730
C***********************************************************************OPT01740
C                                                                       OPT01750
1140  IF (ANGI.LT.135) GO TO 999                                        OPT01760
      IF (ANGI.GE.135 .AND. ANGI.LT.165) GO TO 1141                     OPT01770
      IF (ANGI.GE.165) GO TO 999                                        OPT01780
C                                                                       OPT01790
C***********************************************************************OPT01800
C                                                                       OPT01810
1200  IF (ANGM.LT.-45) GO TO 999                                        OPT01820
      IF (ANGM.GE.-45 .AND. ANGM.LT.-15) GO TO 1210                     OPT01830
      IF (ANGM.GE.-15 .AND. ANGM.LT.-5) GO TO 1220                      OPT01840
      IF (ANGM.GT.-5) GO TO 999                                         OPT01850
C                                                                       OPT01860
C***********************************************************************OPT01870
C                                                                       OPT01880
1210  IF (ANGI.LT.135) GO TO 999                                        OPT01890
      IF (ANGI.GE.135 .AND. ANGI.LT.165) GO TO 1211                     OPT01900
      IF (ANGI.GT.165) GO TO 999                                        OPT01910
C                                                                       OPT01920
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C*************************************************************************OPT01930
C                                                                         OPT01940
1220  IF (ANGI.LT.135) GO TO 999                                          OPT01950
      IF (ANGI.GE.135 .AND. ANGI.LT.165) GO TO 1221                       OPT01960
      IF (ANGI.GT.165) GO TO 999                                          OPT01970
C                                                                         OPT01980
C*************************************************************************OPT01990
CC                                                                        OPT02000
CC                                                                        OPT02010
C     OPTIMAL GAINS FOR ANGS=0 ANGI=90 ANGM=0 ANGO=60                     OPT02020
1111  F11=0.204226E+04                                                    OPT02030
      F12=-.131298E+00                                                    OPT02040
      F13=-.815389E-22                                                    OPT02050
      F14=0.182125E+01                                                    OPT02060
      F15=0.186264E-01                                                    OPT02070
      F16=0.000000E+00                                                    OPT02080
      F17=0.520824E+02                                                    OPT02090
      F18=-.783169E-40                                                    OPT02100
      F19=0.305267E+01                                                    OPT02110
      F21=-.327388E+01                                                    OPT02120
      F22=0.206725E-03                                                    OPT02130
      F23=-.845929E-15                                                    OPT02140
      F24=0.269129E+02                                                    OPT02150
      F25=0.226333E+00                                                    OPT02160
      F26=0.000000E+00                                                    OPT02170
      F27=0.769628E+03                                                    OPT02180
      F28=-.478709E-34                                                    OPT02190
      F29=0.355655E+02                                                    OPT02200
      F31=-.107285E+03                                                    OPT02210
      F32=0.677435E-02                                                    OPT02220
      F33=0.791307E-16                                                    OPT02230
      F34=0.881931E+03                                                    OPT02240
      F35=0.741688E+01                                                    OPT02250
      F36=0.000000E+00                                                    OPT02260
      F37=0.252206E+05                                                    OPT02270
      F38=0.447798E-35                                                    OPT02280
      F39=0.116548E+04                                                    OPT02290
      GO TO 50                                                            OPT02300
C                                                                         OPT02310
C     ANGS=0 ANGI=120 ANGM=-5 ANGO=60                                     OPT02320
1112  F11=0.204140E+04                                                    OPT02330
      F12=-.131289E+00                                                    OPT02340
      F13=0.184866E-08                                                    OPT02350
      F14=0.244617E+02                                                    OPT02360
      F15=0.539820E-01                                                    OPT02370
      F16=0.000000E+00                                                    OPT02380
      F17=0.112314E+03                                                    OPT02390
      F18=0.464785E-24                                                    OPT02400
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      F19=0.134149E+02                                                  OPT02410
      F21=-.533123E+02                                                  OPT02420
      F22=0.303621E-02                                                  OPT02430
      F23=0.134458E-07                                                  OPT02440
      F24=0.517502E+04                                                  OPT02450
      F25=0.487442E+01                                                  OPT02460
      F26=0.000000E+00                                                  OPT02470
      F27=0.943919E+04                                                  OPT02480
      F28=0.178897E-23                                                  OPT02490
      F29=0.118859E+04                                                  OPT02500
      F31=-.127164E+03                                                  OPT02510
      F32=0.778576E-02                                                  OPT02520
      F33=0.124747E-07                                                  OPT02530
      F34=0.136002E+05                                                  OPT02540
      F35=0.124995E+02                                                  OPT02550
      F36=0.000000E+00                                                  OPT02560
      F37=0.239397E+05                                                  OPT02570
      F38=0.536589E-23                                                  OPT02580
      F39=0.304791E+04                                                  OPT02590
      GO TO 50                                                          OPT02600
C                                                                       OPT02610
C     ANGS=0 ANGI=120 ANGM=-10 ANGO=60                                  OPT02620
1121  F11=0.790490E+04                                                  OPT02630
      F12=-.196716E+01                                                  OPT02640
      F13=-.714125E-21                                                  OPT02650
      F14=0.222004E+02                                                  OPT02660
      F15=0.247746E-01                                                  OPT02670
      F16=0.000000E+00                                                  OPT02680
      F17=0.378681E+02                                                  OPT02690
      F18=-.748735E-38                                                  OPT02700
      F19=0.555408E+01                                                  OPT02710
      F110=-.262554E+03                                                 OPT02720
      F21=-.332298E+03                                                  OPT02730
      F22=0.723837E-01                                                  OPT02740
      F23=-.445311E-15                                                  OPT02750
      F24=0.368703E+04                                                  OPT02760
      F25=0.227908E+01                                                  OPT02770
      F26=0.000000E+00                                                  OPT02780
      F27=0.628910E+04                                                  OPT02790
      F28=-.761346E-32                                                  OPT02800
      F29=0.376893E+03                                                  OPT02810
      F210=0.797282E+01                                                 OPT02820
      F31=-.848588E+03                                                  OPT02830
      F32=0.184846E+00                                                  OPT02840
      F33=0.534540E-15                                                  OPT02850
      F34=0.941557E+04                                                  OPT02860
      F35=0.582010E+01                                                  OPT02870
      F36=0.000000E+00                                                  OPT02880
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      F37=0.160605E+05                                                  OPT02890
      F38=0.913901E-32                                                  OPT02900
      F39=0.962472E+03                                                  OPT02910
      F310=0.203602E+02                                                 OPT02920
      GO TO 50                                                          OPT02930
C                                                                       OPT02940
C     ANGS=0 ANGI=90 ANGM=-10 ANGO=60                                   OPT02950
1122  F11=0.790346E+04                                                  OPT02960
      F12=-.196704E+01                                                  OPT02970
      F13=0.301838E-07                                                  OPT02980
      F14=-.142808E+03                                                  OPT02990
      F15=0.291412E-01                                                  OPT03000
      F16=-.117578E-24                                                  OPT03010
      F17=0.705603E+02                                                  OPT03020
      F18=-.200938E-24                                                  OPT03030
      F19=0.634065E+01                                                  OPT03040
      F110=-.262536E+03                                                 OPT03050
      F21=-.563134E+02                                                  OPT03060
      F22=0.119556E-01                                                  OPT03070
      F23=0.987899E-08                                                  OPT03080
      F24=0.130813E+02                                                  OPT03090
      F25=0.340290E+00                                                  OPT03100
      F26=-.470838E-26                                                  OPT03110
      F27=0.114722E+04                                                  OPT03120
      F28=0.148104E-25                                                  OPT03130
      F29=0.534212E+02                                                  OPT03140
      F210=0.129020E+01                                                 OPT03150
      F31=-.122608E+04                                                  OPT03160
      F32=0.260306E+00                                                  OPT03170
      F33=0.214697E-06                                                  OPT03180
      F34=0.286723E+03                                                  OPT03190
      F35=0.740904E+01                                                  OPT03200
      F36=-.103164E-24                                                  OPT03210
      F37=0.249778E+05                                                  OPT03220
      F38=0.318815E-24                                                  OPT03230
      F39=0.116312E+04                                                  OPT03240
      F310=0.280912E+02                                                 OPT03250
      GO TO 50                                                          OPT03260
C                                                                       OPT03270
C                                                                       OPT03280
C     ANGS=-30 ANGI=150 ANGM=-30 ANGO=60                                OPT03290
1211  F11=0.178657E+04                                                  OPT03300
      F12=-.998658E-01                                                  OPT03310
      F13=0.248111E-08                                                  OPT03320
      F14=0.756883E+03                                                  OPT03330
      F15=0.589541E+00                                                  OPT03340
      F16=0.000000E+00                                                  OPT03350
      F17=0.114030E+04                                                  OPT03360
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      F18=0.236869E-24                                                  OPT03370
      F19=0.936273E+02                                                  OPT03380
      F21=0.104871E+04                                                  OPT03390
      F22=0.194154E+00                                                  OPT03400
      F23=0.990552E-08                                                  OPT03410
      F24=0.843671E+04                                                  OPT03420
      F25=0.310764E+01                                                  OPT03430
      F26=0.000000E+00                                                  OPT03440
      F27=0.439299E+04                                                  OPT03450
      F28=-.119553E-24                                                  OPT03460
      F29=0.495943E+03                                                  OPT03470
      F31=0.708747E+04                                                  OPT03480
      F32=-.167420E+00                                                  OPT03490
      F33=0.109366E-07                                                  OPT03500
      F34=0.884347E+04                                                  OPT03510
      F35=0.414295E+01                                                  OPT03520
      F36=0.000000E+00                                                  OPT03530
      F37=0.646119E+04                                                  OPT03540
      F38=0.623306E-24                                                  OPT03550
      F39=0.659744E+03                                                  OPT03560
      GO TO 50                                                          OPT03570
C                                                                       OPT03580
C     ANGS=-30 ANGI=150 ANGM=-10 ANGO=60                                OPT03590
1221  F11=0.641881E+04                                                  OPT03600
      F12=-.154288E+01                                                  OPT03610
      F13=0.611579E-06                                                  OPT03620
      F14=-.822035E+03                                                  OPT03630
      F15=0.654956E+00                                                  OPT03640
      F16=0.225757E+03                                                  OPT03650
      F17=0.125410E+04                                                  OPT03660
      F18=-.571492E-14                                                  OPT03670
      F19=0.978107E+02                                                  OPT03680
      F110=-.211747E+03                                                 OPT03690
      F21=-.386867E+04                                                  OPT03700
      F22=0.264005E+01                                                  OPT03710
      F23=0.210582E-05                                                  OPT03720
      F24=0.112568E+05                                                  OPT03730
      F25=0.284393E+01                                                  OPT03740
      F26=0.540771E+03                                                  OPT03750
      F27=0.348582E+04                                                  OPT03760
      F28=-.136893E-13                                                  OPT03770
      F29=0.465491E+03                                                  OPT03780
      F210=0.379750E+03                                                 OPT03790
      F31=0.128335E+05                                                  OPT03800
      F32=-.147761E+01                                                  OPT03810
      F33=0.331809E-05                                                  OPT03820
      F34=0.870688E+04                                                  OPT03830
      F35=0.451243E+01                                                  OPT03840
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      F36=-.169267E+04                                                  OPT03850
      F37=0.690758E+04                                                  OPT03860
      F38=0.428491E-13                                                  OPT03870
      F39=0.713575E+03                                                  OPT03880
      F310=-.186004E+03                                                 OPT03890
      GO TO 50                                                          OPT03900
C                                                                       OPT03910
C     ANGS=0 ANGI=120 ANGM=-20 ANGO=60                                  OPT03920
1131  F11=0.811671E+04                                                  OPT03930
      F12=-.196614E+01                                                  OPT03940
      F13=-.333818E-07                                                  OPT03950
      F14=0.188933E+04                                                  OPT03960
      F15=0.483956E-01                                                  OPT03970
      F16=0.000000E+00                                                  OPT03980
      F17=-.139207E+04                                                  OPT03990
      F18=0.140756E-23                                                  OPT04000
      F19=0.108259E+02                                                  OPT04010
      F110=-.262390E+03                                                 OPT04020
      F21=-.633443E+03                                                  OPT04030
      F22=0.143016E+00                                                  OPT04040
      F23=-.376307E-08                                                  OPT04050
      F24=0.396744E+04                                                  OPT04060
      F25=0.231410E+01                                                  OPT04070
      F26=0.000000E+00                                                  OPT04080
      F27=0.595382E+04                                                  OPT04090
      F28=0.129334E-23                                                  OPT04100
      F29=0.381170E+03                                                  OPT04110
      F210=0.157266E+02                                                 OPT04120
      F31=-.158742E+04                                                  OPT04130
      F32=0.358846E+00                                                  OPT04140
      F33=-.974890E-08                                                  OPT04150
      F34=0.997180E+04                                                  OPT04160
      F35=0.580637E+01                                                  OPT04170
      F36=0.000000E+00                                                  OPT04180
      F37=0.149255E+05                                                  OPT04190
      F38=0.324027E-23                                                  OPT04200
      F39=0.956405E+03                                                  OPT04210
      F310=0.394602E+02                                                 OPT04220
      GO TO 50                                                          OPT04230
C                                                                       OPT04240
C     ANGS=0 ANGI=150 ANGM=-20 ANGO=60                                  OPT04250
1132  F11=0.759289E+04                                                  OPT04260
      F12=-.195657E+01                                                  OPT04270
      F13=0.870930E-08                                                  OPT04280
      F14=-.874970E+03                                                  OPT04290
      F15=0.965528E-01                                                  OPT04300
      F16=-.807894E-23                                                  OPT04310
      F17=0.867856E+03                                                  OPT04320
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      F18=-.131746E-24                                                  OPT04330
      F19=0.171856E+02                                                  OPT04340
      F110=-.261498E+03                                                 OPT04350
      F21=-.331499E+04                                                  OPT04360
      F22=0.472346E+00                                                  OPT04370
      F23=0.633018E-07                                                  OPT04380
      F24=0.348746E+04                                                  OPT04390
      F25=0.317480E+01                                                  OPT04400
      F26=-.549204E-24                                                  OPT04410
      F27=0.967072E+04                                                  OPT04420
      F28=0.226133E-23                                                  OPT04430
      F29=0.521581E+03                                                  OPT04440
      F210=0.609826E+02                                                 OPT04450
      F31=-.159741E+04                                                  OPT04460
      F32=-.682238E-01                                                  OPT04470
      F33=0.844020E-07                                                  OPT04480
      F34=0.497586E+04                                                  OPT04490
      F35=0.439744E+01                                                  OPT04500
      F36=-.181799E-23                                                  OPT04510
      F37=0.131649E+05                                                  OPT04520
      F38=0.309438E-23                                                  OPT04530
      F39=0.722932E+03                                                  OPT04540
      F310=-.120680E+02                                                 OPT04550
      GO TO 50                                                          OPT04560
C                                                                       OPT04570
C     ANGS=0 ANGI=150 ANGM=-30 ANGO=60                                  OPT04580
1141  F11=0.790298E+04                                                  OPT04590
      F12=-.196635E+01                                                  OPT04600
      F13=0.226082E-08                                                  OPT04610
      F14=0.696921E+02                                                  OPT04620
      F15=0.446845E-01                                                  OPT04630
      F16=-.330942E-04                                                  OPT04640
      F17=0.364668E+02                                                  OPT04650
      F18=0.205154E-20                                                  OPT04660
      F19=0.996732E+01                                                  OPT04670
      F110=-.262424E+03                                                 OPT04680
      F21=-.821164E+03                                                  OPT04690
      F22=0.181858E+00                                                  OPT04700
      F23=-.929652E-08                                                  OPT04710
      F24=0.915014E+04                                                  OPT04720
      F25=0.320679E+01                                                  OPT04730
      F26=0.104216E+04                                                  OPT04740
      F27=0.467566E+04                                                  OPT04750
      F28=-.623528E-13                                                  OPT04760
      F29=0.525775E+03                                                  OPT04770
      F210=0.199699E+02                                                 OPT04780
      F31=-.117923E+04                                                  OPT04790
      F32=0.248020E+00                                                  OPT04800
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      F33=-.184865E-07                                                  OPT04810
      F34=0.121731E+05                                                  OPT04820
      F35=0.437114E+01                                                  OPT04830
      F36=-.234501E+04                                                  OPT04840
      F37=0.662218E+04                                                  OPT04850
      F38=0.140303E-12                                                  OPT04860
      F39=0.716267E+03                                                  OPT04870
      F310=0.272052E+02                                                 OPT04880
      GO TO 50                                                          OPT04890
C                                                                       OPT04900
C                                                                       OPT04910
C*************************************************************************OPT04920
C*************************************************************************OPT04930
C                                                                       OPT04940
444   IF (ANGO.LT.45) GO TO 998                                         OPT04950
      IF (ANGO.GT.45 .AND. ANGO.LE.75) GO TO 998                        OPT04960
      IF (ANGO.GT.75) GO TO 998                                         OPT04970
C                                                                       OPT04980
C*************************************************************************OPT04990
9000  IF (ANGS.LT.-15) GO TO 998                                        OPT05000
      IF (ANGS.GE.-15 .AND. ANGS.LT.15) GO TO 9100                      OPT05010
      IF (ANGS.GE.15) GO TO 998                                         OPT05020
C*************************************************************************OPT05030
9100  IF (ANGM.LT.-45) GO TO 998                                        OPT05040
      IF (ANGM.GE.-45 .AND. ANGM.LT.-15) GO TO 9110                     OPT05050
      IF (ANGM.GE.-15) GO TO 998                                        OPT05060
C*************************************************************************OPT05070
9110  IF (ANGI.LT.135) GO TO 998                                        OPT05080
      IF (ANGI.GE.135 .AND. ANGI.LT.165) GO TO 9111                     OPT05090
      IF (ANGI.GE.165) GO TO 998                                        OPT05100
C*************************************************************************OPT05110
C                                                                       OPT05120
C     ANGS=0 ANGI=150 ANGM=-30 ANGO=60                                  OPT05130
9111  F11=0.790201E+04                                                  OPT05140
      F12=-.196593E+01                                                  OPT05150
      F13=-.101725E-07                                                  OPT05160
      F14=0.811865E+02                                                  OPT05170
      F15=0.519686E-01                                                  OPT05180
      F16=-.729663E-04                                                  OPT05190
      F17=0.405932E+02                                                  OPT05200
      F18=-.581363E-20                                                  OPT05210
      F19=0.115724E+02                                                  OPT05220
      F110=-.262358E+03                                                 OPT05230
      F21=-.962277E+03                                                  OPT05240
      F22=0.213312E+00                                                  OPT05250
      F23=0.564133E-07                                                  OPT05260
      F24=0.929287E+04                                                  OPT05270
      F25=0.324487E+01                                                  OPT05280
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      F26=0.127486E+04                                                  OPT05290
      F27=0.452784E+04                                                  OPT05300
      F28=-.208221E-13                                                  OPT05310
      F29=0.530443E+03                                                  OPT05320
      F210=0.233975E+02                                                 OPT05330
      F31=-.135845E+04                                                  OPT05340
      F32=0.283965E+00                                                  OPT05350
      F33=0.712590E-07                                                  OPT05360
      F34=0.120383E+05                                                  OPT05370
      F35=0.431677E+01                                                  OPT05380
      F36=-.293971E+04                                                  OPT05390
      F37=0.629262E+04                                                  OPT05400
      F38=0.480136E-13                                                  OPT05410
      F39=0.705153E+03                                                  OPT05420
      F310=0.311039E+02                                                 OPT05430
      GO TO 50                                                          OPT05440
C                                                                       OPT05450
CC                                                                      OPT05460
CC    CALCULATE THE VALUE OF THE PITCH INPUT                            OPT05470
C                                                                       OPT05480
50    IF (P.LT.1.5707) THEN                                             OPT05490
         P=8*T*K                                                        OPT05500
      ELSE                                                              OPT05510
         P=1.570796327                                                  OPT05520
      END IF                                                            OPT05530
C                                                                       OPT05540
CC                                                                      OPT05550
CC    CALCULATE VARIABLE GAIN VALUES FROM X(K):                         OPT05560
C                                                                       OPT05570
      CI=COS(X3(K))                                                     OPT05580
      SI=SIN(X3(K))                                                     OPT05590
C                                                                       OPT05600
      CM=COS(X6(K))                                                     OPT05610
      SM=SIN(X6(K))                                                     OPT05620
C                                                                       OPT05630
      CS=COS(X8(K))                                                     OPT05640
      SS=SIN(X8(K))                                                     OPT05650
C                                                                       OPT05660
      CO=COS(X9(K))                                                     OPT05670
      SO=SIN(X9(K))                                                     OPT05680
C                                                                       OPT05690
      Z=SM*SI                                                           OPT05700
C                                                                       OPT05710
C                                                                       OPT05720
      JM=(0.07781+0.04949*CI**2)*192                                    OPT05730
      JO=(0.09318+0.02687*CM**2+0.04428*CM*2*SI**2)*192                 OPT05740
C*****************************************************************    OPT05750
C                                                                       OPT05760
```

```
      IF (TEST.EQ.1) R=PI                                                    OPT05770
C                                                                            OPT05780
CC    CALCULATE CONTROL INPUTS U1, U2 AND U3                                 OPT05790
C                                                                            OPT05800
      U1=-(F11*X1(K)+F12*X2(K)+F13*(X3(K)-R)+F14*X4(K)+F15*X5(K)             OPT05810
     2+F16*X6(K)+F17*X7(K)+F18*X8(K)+F19*X10(K)+F110*X11(K))                 OPT05820
      U2=-(F21*X1(K)+F22*X2(K)+F23*(X3(K)-R)+F24*X4(K)+F25*X5(K)             OPT05830
     2+F26*X6(K)+F27*X7(K)+F28*X8(K)+F29*X10(K)+F210*X11(K))                 OPT05840
      U3=-(F31*X1(K)+F32*X2(K)+F33*(X3(K)-R)+F34*X4(K)+F35*X5(K)             OPT05850
     2+F36*X6(K)+F37*X7(K)+F38*X8(K)+F39*X10(K)+F310*X11(K))                 OPT05860
C                                                                            OPT05870
CC                                                                           OPT05880
CC    CALCULATE THE K+1 VALUE OF EACH VARIABLE                               OPT05890
CC                                                                           OPT05900
CC                                                                           OPT05910
C*************************************************************************OPT05920
CC                                                                           OPT05930
C                                                                            OPT05940
      X1(K+1)=X1(K)+T*0.0678345*U1                                           OPT05950
      X2(K+1)=X2(K)+T*GAIN*(-CS*X1(K)+CI*SS*X4(K)-SI*CM*SS*X7(K)-            OPT05960
     2Z*SO*SS*P)                                                             OPT05970
      X3(K+1)=X3(K)+T*(X1(K)-SM*X7(K)+CM*SO*P)                               OPT05980
      X4(K+1)=X4(K)+T*1/JM*U2                                                OPT05990
      X5(K+1)=X5(K)+T*GAIN*(-CI*CS*X4(K)+CS*SI*CM*X7(K)-                     OPT06000
     2SS*X1(K)+Z*SO*CS*P)                                                    OPT06010
      X6(K+1)=X6(K)+T*(X4(K)-CO*P)                                           OPT06020
      X7(K+1)=X7(K)+T*(1/JO*U3-SM/JO*U1)                                     OPT06030
      X8(K+1)=X8(K)+T*(-CI*CM*X7(K)-SI*X4(K)-SO*CI*SM*P)                     OPT06040
      X9(K+1)=X9(K)+T*X7(K)                                                  OPT06050
      X10(K+1)=X10(K)+T*X5(K)                                                OPT06060
      X11(K+1)=X11(K)+T*X2(K)                                                OPT06070
C                                                                            OPT06080
CC                                                                           OPT06090
CC                                                                           OPT06100
C                                                                            OPT06110
      IF (K.EQ.2800) GO TO 777                                               OPT06120
10    CONTINUE                                                               OPT06130
C                                                                            OPT06140
CC                                                                           OPT06150
CC                                                                           OPT06160
C                                                                            OPT06170
998   TTT=T*K                                                                OPT06180
      CALL FRTCMS('CLRSCRN ')                                                OPT06190
      PRINT *,' '                                                            OPT06200
      PRINT *,' '                                                            OPT06210
      PRINT *,' '                                                            OPT06220
      PRINT *,'      OPTIMAL FEEDBACK GAINS HAVE NOT BEEN COMPUTED '         OPT06230
      PRINT *,'      FOR THE CURRENT GIMBAL ANGLES. NULLING SIN(AI)'         OPT06240
```

```
      PRINT *,'         ANGS=',ANGS                                          OPT06250
      PRINT *,'         ANGI=',ANGI                                          OPT06260
      PRINT *,'         ANGM=',ANGM                                          OPT06270
      PRINT *,'         ANGO=',ANGO                                          OPT06280
      PRINT *,' '                                                            OPT06290
      PRINT *,'         K=',K                                                OPT06300
      PRINT *,'         T=',TTT                                              OPT06310
      PRINT *,'         P=',P                                                OPT06320
      GO TO 777                                                              OPT06330
C                                                                            OPT06340
999   TTT=T*K                                                                OPT06350
      CALL FRTCMS('CLRSCRN ')                                                OPT06360
      PRINT *,' '                                                            OPT06370
      PRINT *,' '                                                            OPT06380
      PRINT *,' '                                                            OPT06390
      PRINT *,'       OPTIMAL FEEDBACK GAINS HAVE NOT BEEN COMPUTED '        OPT06400
      PRINT *,'       FOR THE CURRENT GIMBAL ANGLES. NULLING SIN(AM)'        OPT06410
      PRINT *,'         ANGS=',ANGS                                          OPT06420
      PRINT *,'         ANGI=',ANGI                                          OPT06430
      PRINT *,'         ANGM=',ANGM                                          OPT06440
      PRINT *,'         ANGO=',ANGO                                          OPT06450
      PRINT *,' '                                                            OPT06460
      PRINT *,'         K=',K                                                OPT06470
      PRINT *,'         T=',TTT                                              OPT06480
      PRINT *,'         P=',P                                                OPT06490
      GO TO 777                                                              OPT06500
C                                                                            OPT06510
777   DO 80 I=1,K,25                                                         OPT06520
C                                                                            OPT06530
      TT=T*I                                                                 OPT06540
C                                                                            OPT06550
C     WRITE GYRO ERRORS TO ERR DATA FILE                                     OPT06560
C                                                                            OPT06570
      UGSGF=X2(I)                                                            OPT06580
      VGSGF=X5(I)                                                            OPT06590
C                                                                            OPT06600
C     WRITE GIMBAL ANGLES TO ANG DATA FILE                                   OPT06610
C                                                                            OPT06620
      ANGI=X3(I)*180/PI                                                      OPT06630
      ANGM=X6(I)*180/PI                                                      OPT06640
      ANGS=X8(I)*180/PI                                                      OPT06650
      ANGO=X9(I)*180/PI                                                      OPT06660
      WRITE (7,30) TT,UGSGF,VGSGF,ANGI,ANGM                                  OPT06670
30    FORMAT(5E14.6)                                                         OPT06680
      WRITE (8,40) TT,ANGI,ANGM,ANGS,ANGO                                    OPT06690
40    FORMAT (5E14.6)                                                        OPT06700
80    CONTINUE                                                               OPT06710
C                                                                            OPT06720
```

```
C
      STOP                                                              OPT06730
      END                                                               OPT06740
                                                                        OPT06750
```

APPENDIX D TABULATED DATA

TIME(SEC)	UGSGF	VGSGF	INNER ANGLE	MIDDLE ANGLE
0.333333E-03	0.000000E+00	0.000000E+00	0.900001E+02	0.000000E+00
0.866666E-02	-0.277583E-04	-0.145522E-01	0.900143E+02	-0.846295E-02
0.170000E-01	-0.103715E-02	-0.133000E+00	0.900559E+02	-0.332002E-01
0.253333E-01	-0.662004E-02	-0.385341E+00	0.901248E+02	-0.742088E-01
0.336666E-01	-0.219164E-01	-0.762853E+00	0.902211E+02	-0.131484E+00
0.419999E-01	-0.524359E-01	-0.126719E+01	0.903448E+02	-0.205019E+00
0.503333E-01	-0.103713E+00	-0.189803E+01	0.904958E+02	-0.294804E+00
0.586666E-01	-0.181147E+00	-0.265487E+01	0.906741E+02	-0.400826E+00
0.669999E-01	-0.289998E+00	-0.353736E+01	0.908799E+02	-0.523072E+00
0.753332E-01	-0.435436E+00	-0.454502E+01	0.911130E+02	-0.661523E+00
0.836666E-01	-0.622578E+00	-0.567730E+01	0.913735E+02	-0.816158E+00
0.919999E-01	-0.856513E+00	-0.693363E+01	0.916613E+02	-0.986950E+00
0.100333E+00	-0.114231E+01	-0.831339E+01	0.919766E+02	-0.117387E+01
0.108667E+00	-0.148502E+01	-0.981594E+01	0.923193E+02	-0.137688E+01
0.117000E+00	-0.188966E+01	-0.114406E+02	0.926894E+02	-0.159594E+01
0.125333E+00	-0.236123E+01	-0.131867E+02	0.930869E+02	-0.183100E+01
0.133667E+00	-0.290470E+01	-0.150534E+02	0.935120E+02	-0.208201E+01
0.142000E+00	-0.352500E+01	-0.170402E+02	0.939646E+02	-0.234891E+01
0.150333E+00	-0.422702E+01	-0.191464E+02	0.944448E+02	-0.263162E+01
0.158666E+00	-0.501565E+01	-0.213711E+02	0.949527E+02	-0.293006E+01
0.167000E+00	-0.589571E+01	-0.237139E+02	0.954882E+02	-0.324414E+01
0.175333E+00	-0.687201E+01	-0.261741E+02	0.960516E+02	-0.357375E+01
0.183666E+00	-0.794933E+01	-0.287511E+02	0.966428E+02	-0.391879E+01
0.192000E+00	-0.913241E+01	-0.314444E+02	0.972620E+02	-0.427913E+01
0.200333E+00	-0.104209E+02	-0.304900E+02	0.979065E+02	-0.465295E+01
0.208666E+00	-0.116269E+02	-0.216803E+02	0.985557E+02	-0.502813E+01
0.217000E+00	0.668379E+02	-0.204485E+02	0.992014E+02	-0.540227E+01
0.225333E+00	0.406588E+01	-0.220079E+02	0.998553E+02	-0.577529E+01
0.233666E+00	-0.734333E+01	-0.224933E+02	0.100507E+03	-0.614706E+01
0.242000E+00	0.145961E+01	-0.223943E+02	0.101159E+03	-0.651756E+01
0.250333E+00	-0.592441E-02	-0.221652E+02	0.101813E+03	-0.688676E+01
0.258666E+00	-0.628444E+00	-0.221838E+02	0.102467E+03	-0.725461E+01
0.267000E+00	-0.270133E+00	-0.221181E+02	0.103122E+03	-0.762105E+01
0.275333E+00	-0.301474E+00	-0.220531E+02	0.103778E+03	-0.798606E+01
0.283666E+00	-0.332833E+00	-0.220001E+02	0.104436E+03	-0.834959E+01
0.292000E+00	-0.300393E+00	-0.141415E+02	0.105095E+03	-0.871125E+01
0.300333E+00	0.152929E+01	0.629700E+02	0.105755E+03	-0.906673E+01
0.308666E+00	0.128248E+00	-0.390406E+02	0.106415E+03	-0.942558E+01
0.317000E+00	-0.597174E+00	-0.199967E+02	0.107077E+03	-0.977518E+01
0.325333E+00	-0.173000E+00	-0.180456E+02	0.107741E+03	-0.101213E+02
0.333666E+00	-0.219891E+00	-0.198598E+02	0.108405E+03	-0.104636E+02
0.342000E+00	-0.252207E+00	-0.184565E+02	0.109071E+03	-0.108017E+02
0.350333E+00	-0.225724E+00	-0.180980E+02	0.109737E+03	-0.111359E+02
0.358666E+00	-0.221078E+00	-0.176057E+02	0.110405E+03	-0.114662E+02
0.367000E+00	-0.216717E+00	-0.170973E+02	0.111075E+03	-0.117927E+02
0.375333E+00	-0.209937E+00	-0.166263E+02	0.111745E+03	-0.121154E+02
0.383666E+00	-0.204042E+00	-0.161582E+02	0.112417E+03	-0.124343E+02

0.392000E+00	-0.198381E+00	-0.157024E+02	0.113089E+03	-0.127495E+02
0.400333E+00	-0.192729E+00	-0.152567E+02	0.113764E+03	-0.130610E+02
0.408666E+00	-0.187223E+00	-0.148204E+02	0.114439E+03	-0.133689E+02
0.416999E+00	-0.181840E+00	-0.143931E+02	0.115116E+03	-0.136733E+02
0.425333E+00	-0.176559E+00	-0.139742E+02	0.115793E+03	-0.139741E+02
0.433666E+00	-0.171380E+00	-0.135633E+02	0.116472E+03	-0.142715E+02
0.441999E+00	-0.166297E+00	-0.131597E+02	0.117153E+03	-0.145654E+02
0.450333E+00	-0.161302E+00	-0.127630E+02	0.117834E+03	-0.148559E+02
0.458666E+00	-0.707598E+02	0.246526E+02	0.118521E+03	-0.151420E+02
0.466999E+00	-0.677763E+02	-0.567121E+01	0.119205E+03	-0.154269E+02
0.475333E+00	0.141161E+02	-0.151967E+02	0.119885E+03	-0.157077E+02
0.483666E+00	0.202555E+01	-0.883315E+01	0.120572E+03	-0.159851E+02
0.491999E+00	-0.322556E+01	-0.112647E+02	0.121260E+03	-0.162597E+02
0.500333E+00	-0.180843E+00	-0.102183E+02	0.121949E+03	-0.165310E+02
0.508666E+00	-0.468412E+00	-0.991344E+01	0.122639E+03	-0.167993E+02
0.516999E+00	-0.768435E+00	-0.963894E+01	0.123331E+03	-0.170646E+02
0.525333E+00	-0.671265E+00	-0.929060E+01	0.124023E+03	-0.173270E+02
0.533666E+00	-0.686222E+00	-0.896907E+01	0.124717E+03	-0.175863E+02
0.541999E+00	-0.713157E+00	-0.864934E+01	0.125411E+03	-0.178427E+02
0.550333E+00	-0.721314E+00	-0.833055E+01	0.126107E+03	-0.180962E+02
0.558666E+00	-0.731350E+00	-0.801416E+01	0.126805E+03	-0.183469E+02
0.566999E+00	-0.740838E+00	-0.769972E+01	0.127503E+03	-0.185946E+02
0.575333E+00	-0.748012E+00	-0.738669E+01	0.128202E+03	-0.188396E+02
0.583666E+00	-0.753631E+00	-0.707485E+01	0.128903E+03	-0.190817E+02
0.591999E+00	-0.757636E+00	-0.676391E+01	0.129604E+03	-0.193211E+02
0.600333E+00	-0.759911E+00	-0.645356E+01	0.130307E+03	-0.195578E+02
0.608666E+00	-0.760450E+00	-0.614349E+01	0.131010E+03	-0.197917E+02
0.616999E+00	-0.759221E+00	-0.583340E+01	0.131715E+03	-0.200229E+02
0.625333E+00	-0.756183E+00	-0.552299E+01	0.132421E+03	-0.202515E+02
0.633666E+00	-0.751300E+00	-0.521195E+01	0.133127E+03	-0.204774E+02
0.641999E+00	-0.744536E+00	-0.489998E+01	0.133835E+03	-0.207008E+02

LIST OF REFERENCES

1. C. S. Draper Laboratory Report CSDL-P-1858, A Discrete Time Cost-Optimal Regulator for the All-Attitude Gimbal Assembly, by G. Lauro, February 1984

2. Martinage, L., Four Gimbal IMU Model, C. S. Draper Laboratory memorandum 20E:73:82.

3. C. S. Draper Laboratory Report, Electronics Design Review, 18 December 1985.

4. Kwakernaak, H. and Sivan, R., Linear Optimal Control Systems, John Wiley & Sons, Inc., 1972.

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